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IN THE UNITED STATES PATENT AND TRADEMARK OFFICE
BEFORE THE BOARD OF PATENT APPEALS AND INTERFERENCES

First Named

Inventor : Pradeep Kumar Subrahmanyam

Appeal No.:

Appln. No. : 09/651,046

Group Art Unit: 2128

Filed : August 30, 2000

Examiner: H. M. Jones


For : PROCESS FOR DESIGNING AN OPTIMAL
VIBRATION ISOLATION MOUNT FOR A
DISC DRIVE

Docket No. : S01.12-0632/STL 9334.00

BRIEF FOR APPELLANT

Mail Stop Appeal Brief-Patents
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ALEXANDRIA, VA 22313-1450, THIS

18th DAY OF July, 20 05

PATENT ATTORNEY

Sir:

This is an appeal from an Office Action dated March 9, 2005 in which claims 1, 2 and 5 were finally rejected.

REAL PARTY IN INTEREST

Seagate Technology LLC, a limited liability corporation organized under the laws of the state of Delaware, and having offices at 920 Disc Drive, Scotts Valley, California 95066, has acquired the entire right, title and interest in and to the invention, the application, and any and all patents to be obtained therefor, as set forth in the Assignment filed with the patent application and recorded on Reel 011063, frame 0119.

RELATED APPEALS AND INTERFERENCES

There are no known related appeals or interferences regarding the present appeal.

STATUS OF THE CLAIMS

- I. Total number of claims in the application.
Claims in the application are: 8
- II. Status of all the claims.
 - A. Claims canceled: 0
 - B. Claims withdrawn but not canceled: 0
 - C. Claims pending: 8
 - D. Claims allowed: 0
 - E. Claims rejected: 3
 - F. Claims objected to: 5
- III. Claims on appeal
The claims on appeal are: 1-8

STATUS OF AMENDMENTS

An Amendment was filed after the mailing of the final rejection. The Amendment was in response to the 35 USC § 112, first paragraph and 35 USC § 112, second paragraph rejections. The Examiner indicated, in the Advisory Action, that the Amendment was entered. Therefore, the Appellant believes the § 112 rejection was withdrawn.

SUMMARY OF CLAIMED SUBJECT MATTER

I. INTRODUCTION

The present invention relates to a disc drive. More particularly, the present invention relates to a process of designing an optimal passive damping mount for a disc drive.

II. BRIEF BACKGROUND

There are two types of disturbances that affect operation of disc drives. These two types of disturbances are internal disturbances and external disturbances. Examples of internal disturbances include windage created by the rotating disc and encountered by the head/arm assembly, noise in the position error signal (PES), structural resonance of the disc drive, motion of the actuator

assembly, imbalance of the discs, written-in run out in the servo patterns due to disturbances during writing of the servo patterns, unmodeled dynamics and non-linearities, such as pivot bearing friction. Examples of external disturbances include environmental (seismic) vibration and external reactions to internal disturbances.

When a disc drive is subjected to disturbances, head/disc tracking is adversely affected. Generally, the heads and the data tracks on the disc have some defined width that is large enough to assure that disturbances do not materially affect the performance of the disc drive. However, the defined width of the data tracks limit the maximum areal data density of the disc. Typically, passive damping mounts are mounted to the bottom of the disc drive to isolate disturbances. As the stiffness of the damping mounts decreases, better attenuation of seismic vibration is obtained. However, soft isolation mounts cannot attenuate internally generated disturbances. Generally, internally generated disturbances can be attenuated by hard or stiff mounts. Therefore, external and internal vibrations have conflicting effects when using passive damping mounts. Attenuation of one source of disturbance by passive mounts leads to amplification of the other source.

III. THE PRESENT INVENTION

A. INDEPENDENT CLAIM 1

Independent claim 1 of the present invention provides a process of designing an optimal vibration mount for a disc drive that eliminates the conflicting effects of a mount that is too stiff or a mount that is too soft. The process includes the steps of computing (202) an external disturbance model for the disc drive and computing (202) an internal disturbance model for the disc drive (see the Specification on page 5, line 25 through page 7, line 4 and page 9, line 15 through line 19). The process includes defining (200) an inertia matrix for the disc drive (see Specification on page 9, lines 15 through 16). The process includes defining (204) a state estimator based on the inertia matrix and external and internal disturbance models to minimize a defined norm of a state estimation error (see the Specification on page 7, line 5 through page 8, line 4 and page 9, line 20-25). The process includes calculating (206, 208) the gain of the state estimator as a solution to a filter algebraic Riccati equation (see the Specification on page 8, line 5 through page 9, line 12 and page 10, line 1 through line 6). The process also includes defining (210) optimal mount damping

and stiffness parameters based on the calculated state estimator gain (see the Specification on page 10, lines 6 through 9).

GROUND OF REJECTION TO BE REVIEWED ON APPEAL

- I. Whether claims 1-2 are unpatentable over Kim et al. (See Appendix B, Exhibit A) in view of Brown et al. (See Appendix B, Exhibit B) or Lee (See Appendix B, Exhibit C).
- II. Whether claim 5 is unpatentable over Kim et al. in view of Brown et al. and further in view of Reif et al. (See Appendix B, Exhibit D) or Shah et al. (See Appendix B, Exhibit E).

ARGUMENT

I. INTRODUCTION

The present invention provides a process of designing an optimal vibration mount or optimal passive damping mount for a disc drive. Typically vibration mounts or damping mounts are used to attenuate seismic or external disturbances as well as to attenuate internal disturbances of a disc drive. Soft mounts can attenuate seismic disturbances well, while stiff mounts can attenuate internal disturbances well. These conflicting effects of soft mounts versus stiff mounts causes attenuation of one source of disturbance and amplification of the other source of disturbance. Embodiments of the present invention include steps for designing an optimal mount that can attenuate both internal and external disturbances without producing conflicting effects. Claims 1, 2 and 5 are currently rejected by at least three different references.

II. PROSECUTION OF THE PRESENT APPLICATION

Appellant filed patent application serial number 09/651,046 on August 30, 2000, claiming priority to provisional application serial number 60/162,269 filed October 28, 1999. The original non-provisional application set forth claims 1-8, which are still pending. In an Office Action dated July 12, 2004, claims 1, 2 and 5 were rejected and claims 3-4 and 6-8 were objected to. Appellant responded on October 12, 2004 by way of an Amendment that amended claim 1. Thereafter, Appellant received a Final Office Action mailed March 9, 2005 rejecting claims 1-8 under 35 USC § 112, first and second paragraphs, rejecting claims 1, 2 and 5 under 35 USC § 103(a) and objecting

to claims 3-4 and 6-8. Appellant responded to the Final Office Action on May 5, 2005 by way of an Amendment that amended claim 1. The Amendment After Final amended claim 1 back to its original form. The Examiner indicated in the Advisory Action mailed June 3, 2005 that the Amendment After Final was entered. Therefore, the 35 USC § 112 rejection should be considered withdrawn. However, claims 1, 2 and 5 still remain rejected under 35 USC § 103(a) and claims 4-6 and 7-8 still remain objected to.

III. PRIOR ART

Kim et al. (Exhibit A) teaches a control system design method that uses Linear Matrix Inequality (LMI), that replaces or complements the Lyapunov-Riccati equations, to suppress shaft vibration of an industrial motor drive system. The resulting method proposes an H infinity speed controller that uses weights to satisfy tracking and disturbance rejection for an induction motor. The control system design of the Kim et al reference includes the use of three different concepts. These are: a two-mass resonant system, an H infinity generalized plant and a control system design by LMI. The Kim et al. reference validates the proposed controller by comparing it to a conventional state feedback controller using a pole placement method. The Examiner further attached to the Kim et al. reference an Abstract of a different paper that the same author had published. However, the Examiner neither cited this as a reference in the PTO-892 form nor referred to it in any of the mailed Office Actions as a reference. Therefore, although the Appellant acknowledges the attachment to the Kim et al. references, the Appellant is not addressing its existence in the following arguments.

Brown et al. (Exhibit B) teaches the design of a research servowriter to record magnetic position encoding schemes in magnetic disk storage applications and to evaluate disk drive technologies. The servowriter includes signal processing hardware, a programmable "pattern generator" controlled by a supervisory computer and a laser interferometric system to measure the position of the read/write head position to thereby estimate the velocity of the read/write head for implementation of a state-variable-based proportional-integral-derivative controller on a microprocessor. The reference also discusses error sources. Sources of error include environmental factors, such as seismic vibration and humidity, spindle and head suspension jitter and other

external vibrations. The reference suggests selecting in-line heads to reduce tangential jitter, isolating the servowriter from external vibrations by mounting it on a large granite base that floats on air isolation mounts and placing the servowriter in a clean room to prevent environmental contamination.

Lee (Exhibit C) proposes an H infinity model matching algorithm for motor tracking control. The control method of Lee provides improved drive performance and reduced sensitivity of disturbances. The algorithm is based on the discrete Riccati equation and the H infinity interpolation theory. The controller effectively controls the presence of external disturbances.

Reif et al. (Exhibit D) teaches non-linear state observation using a Riccati differential equation to improve the extended Kalman filter. The Reif et al reference applies the non-linear state observation to an induction motor for the estimation of flux and angular velocity. In addition, the Reif et al. reference discusses the proposed non-linear state observation in terms of the robust control theory and H infinity filtering.

Shah et al. (Exhibit E) teaches a method for estimating state variables of non-linear control systems using improved extended Kalman filtering techniques. The improved Kalman filtering techniques include using a continuous-time extended Kalman filter (CTEKF) to estimate all state variables. The CTEKF uses discrete time measurement and integration.

IV. THE REJECTION OF CLAIMS 1, 2 AND 5 SHOULD BE REVERSED

Claims 1, 2 and 5 have been rejected under 35 USC §103(a) with various combinations of the cited references. To establish a *prima facie* case of obviousness, three basic criteria must be met. First, there must be some suggestion or motivation either in the references themselves or in the knowledge generally available to one of ordinary skill in the art, to modify the reference or to combine reference teachings. Second, there must be a reasonable expectation of success. Finally, the prior art reference (or references when combined) must teach or suggest all of the claim limitations. *In re Vaeck*, 20 U.S.P.Q.2d 1438 (Fed. Cir. 1991); M.P.E.P. §2143.

In the Response to Arguments section of the Final Rejection, the Examiner stated that “Applicants have merely paraphrased selected portions of the applied art without addressing the sections of the applied art as indicated in the last Office Action.” It is respectfully pointed out

that in accordance with MPEP § 2142, “The examiner bears the initial burden of factually supporting any *prima facie* case of obviousness. If the examiner does not produce a *prima facie* case, the applicant is under no obligation to submit evidence of non-obviousness.” It is respectfully submitted that where no *prima facie* case of obviousness is substantiated, the Applicant has no burden to reply. However, the Appellant, as will be thoroughly discussed below, has diligently pointed out elements of the claims that are not taught or suggested by the cited references. Furthermore, 37 CFR §1.104(c)(2) states the following:

In rejecting claims for want of novelty or for obviousness, the examiner must cite the best references at his or her command. When a reference is complex or shows or describes inventions other than that claimed by the applicant, the particular art relied on must be designated as nearly as practical. The pertinence of each reference, if not apparent, must be clearly explained and each rejected claim specified.

Not only has the Examiner not satisfied the burden of showing all of the elements of the claims in the cited references, the Examiner has cited references which have features that are not readily apparent as being relevant to elements of the claims. The cited references are complex and are not pertinent to the scope of the present invention.

A. Claims 1 and 2 are Allowable Over the Art of Record.

Sections eight and nine of the Final Rejection indicated that claims 1 and 2 were rejected under 35 USC § 103(a) as being unpatentable over Kim et al. in view of Brown et al. or Lee. Appellant respectfully submits that the evidence of record in the Final Rejection is insufficient to establish a *prima facie* case of obviousness. In particular, it is respectfully submitted that even if the Kim et al. and Brown et al. or Lee references were combined, the combination of cited references simply do not teach or suggest all of the elements recited in claims 1 and 2. In particular, the combination of references fail to teach or suggest:

- computing an external disturbance model for the disc drive
- computing an internal disturbance model for the disc drive
- defining an inertia matrix for the disc drive

- defining a state estimator based on the inertia matrix and external and internal disturbance models to minimize a defined norm of a state estimation error
- calculating the gain of the state estimator as a solution to a filter algebraic Riccati equation
- defining optimal mount damping and stiffness parameters based on the calculated state estimator gain

Before thoroughly discussing each of the above bulleted elements, it should be noted that the rotation drives and drive systems that are referred to in the Kim et al. and Lee references are drives that rotate and include means for giving motion to a machine or part of a machine. The rotation drives and drive systems referred to are not a disc drive as claimed in claim 1. As described in the Specification with reference to FIG. 1, a disc drive is a device for storing data. Although a spindle motor is located in a disc drive for rotating a disc, a disc drive includes many other components for reading and writing data to a disc. These components and their operations are discussed in connection with FIG. 1 of the Specification.

In section nine of the Final Rejection, the Examiner finds that Kim et al. discloses “computing an external disturbance model for a disc drive” and “computing an internal disturbance model for a disc drive” as recited in independent claim 1. It is respectfully submitted that the Kim et al. reference neither discloses modeling external disturbances for a disc drive nor modeling internal disturbances for a disc drive. In fact, the Kim et al. reference does not discuss external disturbances at all and, instead, teaches a method of controlling vibration of a drive shaft. Instead, and as mentioned above, the Kim et al. reference teaches a method of designing a control system for a motor drive system that suppresses shaft vibration. The Specification of the present invention specifically notes that examples of external disturbances for a disc drive are seismic vibrations that are produced external to the disc drive. Motor shaft vibration is not an external disturbance to a drive system. In addition, the method taught in the Kim et al. reference is specific to vibration suppression in an industrial or induction type motor. Kim et al. does not teach or suggest computing a model for any type of disturbance (externally or internally) for a disc drive. In fact, the Kim et al. does not even mention a disc drive. Although shaft vibration can potentially be a source of internal disturbance in a disc drive because a disc drive includes a spindle motor, it is well known, and

discussed in the Background of the Specification, that disc drives include many more sources of internal disturbances that are not considered in the Kim et al. reference.

Furthermore, neither the Brown et al. reference nor the Lee reference correct deficiencies of the Kim et al reference as pertaining to computing “an external disturbance model for a disc drive” and “an internal disturbance model for a disc drive”. Brown et al. does not teach or suggest modeling external disturbances for a disc drive nor modeling internal disturbances for a disc drive. Although Brown et al. identifies error sources for a servo writer, Brown et al. suggests ways of mitigating the identified error sources by selecting in-line heads to reduce tangential jitter, isolating the servowriter from external vibrations by mounting in on a base that floats on air isolation mounts and placing the servowriter in a clean room. Brown et al. does not attempt to model internally or externally derived disturbances for a disc drive. In addition, the Lee reference does not teach or suggest modeling external or internal disturbances for a disc drive. Although Lee identifies load torque on the motor shaft of a motor drive system as being a disturbance to consider, Lee does not disclose modeling disturbances for a disc drive.

In section nine of the Final Rejection, the Examiner finds that Kim et al. defines “an inertia matrix for the disc drive” as recited in independent claim 1. However, It is respectfully submitted that Kim et al. does not teach or suggest such a claim element. Kim et al. discloses a motor inertia value and a load inertia value when providing a two-mass resonant system. Although a disc drive includes a spindle motor, an inertia matrix for a disc drive, in its entirety, is not defined in Kim et al. As discussed in the Specification in reference to FIG. 1, a disc drive includes many more components than just a motor. Such components are not considered by the Kim et al. reference.

Furthermore, neither the Brown et al. reference nor the Lee reference correct deficiencies of the Kim et al reference as pertaining to “defining an inertia matrix for the disc drive”. Neither Brown et al. nor Lee teach or suggest defining an inertia matrix for a disc drive.

In section nine of the Final Rejection, the Examiner finds that Kim et al. defines a “state estimator based on the inertia matrix and external and internal disturbance models” and calculates “the gain of the state estimator as a solution filter algebraic Riccatti equation” as recited in claim 1. It is respectfully submitted that the Kim et al. reference, as discussed above, does not teach or suggest such claim elements. It is believed that the Kim et al. reference does not teach or suggest

defining an inertia matrix or computing external and internal disturbance models for a disc drive. Therefore, Kim et al. could not possibly define a state estimator based on these elements as well as calculating a gain of such a state estimator. The Examiner states that “Kim et al. discloses use of Riccati solutions to the reduction of vibration in rotational drives including inertial matrices for internal disturbances” in section 2-4 of the reference. This statement and the information relayed in sections 2-4 of the reference do not read on these or any of the above listed claim elements.

Furthermore, neither the Brown et al. reference nor the Lee reference correct deficiencies of the Kim et al reference as pertaining to “defining a state estimator based on the inertia matrix and external and internal disturbance models to minimize a defined norm of a state estimation error” and “calculating the gain of the state estimator as a solution to a filter algebraic Riccati equation”. Neither Brown et al. nor Lee teach or suggest defining a state estimator based on the inertia matrix and external and internal disturbance models to minimize a defined norm of a state estimation error or calculating the gain of the state estimator as a solution to a filter algebraic Riccati equation.

In section nine of the Final Rejection, the Examiner finds that Kim et al. defines “optimal mount damping stiffness parameters based on the calculated state estimator gain” as recited in claim 1. It is respectfully submitted that the Kim et al. reference neither discloses defining optimal mount damping stiffness parameters nor defining optimal mount damping stiffness parameters based on the calculated state estimator gain. In fact, the Kim et al. reference does not discuss mounts for a disc drive at all and, instead, teaches a control system for suppressing vibration of a motor shaft. In addition, it is believed that the Kim et al. reference does not teach or suggest defining an inertia matrix, computing external and internal disturbance models for a disc drive, defining a state estimator, or calculating the gain of the state estimator. Therefore, Kim et al. could not possibly calculate any parameter, let alone an optimal mount damping and stiffness parameter, based on the state estimator.

Furthermore, neither the Brown et al. reference nor the Lee reference correct deficiencies of the Kim et al reference as pertaining to “defining optimal mount damping and stiffness parameters based on the calculated state estimator gain”. Although Brown et al. identifies the use of a base that floats on air isolation mounts, Brown et al. does not teach or suggest a method of defining optimal mount damping and stiffness parameters nor teaching or suggesting a method of defining optimal

mount damping and stiffness parameters based on the calculated state estimator gain. Lee does not teach or suggest a disc drive, let alone optimal damping mounts.

In section ten, the Final Rejection conceded that Kim et al. did not disclose taking into account external disturbances in addition to the internal disturbances. The Final Rejection turns to the teachings of Brown et al. or Lee as disclosing external as well as internal disturbances to motor drives. In particular, the Final Rejection found that Brown et al. identifies environmental factors such as seismic vibration, temperature change and steps to contain these error sources are described. The Appellant finds this concession irrelevant and moot in view of the fact that the Examiner has not provided a *prima facie* case of non-obviousness. Specifically, the Examiner has not shown that other elements of the claim (as bulleted above) are found within the combination of references.

In section nine of the Final Rejection, the Examiner finds that Kim et al. calculates “covariance matrix based on the solution to the filter algebraic Riccati equation” and calculates “the state estimator gain based on the covariance matrix” as recited in claim 2. In one aspect, it is respectfully submitted that Kim et al. does not teach or suggest the elements of claim 2. Kim et al. does not disclose the use of a covariance matrix, let alone the use of a covariance matrix to calculate the state estimator gain. In another aspect, claim 2 is allowable over the cited reference as depending from an allowable base claim.

Furthermore, neither the Brown et al. reference nor the Lee reference correct deficiencies of the Kim et al reference with respect to the elements of claim 2. Neither Brown et al. nor Lee teach or suggest calculating a covariance matrix based on the solution to the filter algebraic Riccati equation nor calculating the state estimator gain based on the covariance matrix.

It is respectfully believed that claims 1 and 2 are in condition for allowance. Reversal is respectfully requested.

B. Claim 5 is Allowable Over the Art of Record.

In section fifteen of the Office Action, claim 5 is rejected under 35 USC §103(a) as being unpatentable over Kim et al. in view of Brown et al. and further in view of Reif et al. or Shah et al. Appellant respectfully submits that the evidence of record in the Final Rejection is insufficient to establish a *prima facie* case of obviousness. In particular, it is respectfully submitted that even if the

Kim et al. and Brown et al. and further the Reif et al or Shah et al. references were combined, the combination of cited references simply do not teach or suggest all of the elements recited in claim 5. In particular, the combination of references fail to teach or suggest that the state estimator of claim 1 is a Kalman Filter as claimed in claim 5.


The Examiner concedes, in section sixteen of the Final Rejection, that Kim et al. is silent on whether the state estimator is a Kalman Filter. In section seventeen, the Examiner looks to Reif et al. or Shah et al. as teaching a Kalman Filter. The Appellant finds this concession irrelevant and moot in view of the fact that the Examiner has not provided a prima facie case of non-obviousness related to other elements included in claim 5 and as discussed above with respect to claim 1. In particular, claim 5 is allowable over the cited references as depending on an allowable base claim.

V. CONCLUSION

The cited references in the Final Rejection fail to teach or suggest the feature recited in claims 1, 2 and 5. Accordingly, Apellant respectfully requests that the Board reverse the Examiner and find that claims 1, 2 and 5 are in condition for allowance.

Respectfully submitted,

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LRT/jme

Appendix A: Claims

1. A process of designing an optimal vibration mount for a disc drive comprising steps of:
 - a) computing an external disturbance model for the disc drive;
 - b) computing an internal disturbance model for the disc drive;
 - c) defining an inertia matrix for the disc drive;
 - d) defining a state estimator based on the inertia matrix and external and internal disturbance models to minimize a defined norm of a state estimation error;
 - e) calculating the gain of the state estimator as a solution to a filter algebraic Riccati equation; and
 - f) defining optimal mount damping and stiffness parameters based on the calculated state estimator gain.
2. The process of claim 1, wherein step (e) includes,
 - e1) calculating a covariance matrix based on the solution to the filter algebraic Riccati equation, and
 - e2) calculating the state estimator gain based on the covariance matrix.
3. The process of claim 1, wherein step (e) is performed by
 - (e1) calculating a covariance matrix Σ from a solution of the filter algebraic Riccati equation in the form
$$\begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix} \Sigma + \Sigma \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}' + \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix} \Xi \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix}' - \Sigma \begin{pmatrix} 0 & I \end{pmatrix}' \Theta^{-1} \begin{pmatrix} 0 & I \end{pmatrix} \Sigma = 0,$$
where M is the inertia matrix, Θ is the internal disturbance matrix and Ξ is the external disturbance matrix, and
 - (e2) calculating the state estimator gain H from $H = \Sigma \begin{pmatrix} I & 0 \end{pmatrix}' \Theta^{-1}$.

4. The process of claim 3, wherein step (f) is performed by
 - f1) solving $H = \begin{pmatrix} M^{-1}B \\ M^{-1}K \end{pmatrix}$ for B and K, and
 - f2) setting the optimal mount damping matrix to B and the optimal mount stiffness matrix to K.
5. The process of claim 1 wherein the state estimator is a Kalman filter.
6. The process of claim 5, wherein step e) includes,
 - e1) calculating a covariance matrix based on the solution to the filter algebraic Riccati equation, and
 - e2) calculating the Kalman filter gain based on the covariance matrix and the inertia matrix.
7. The process of claim 6, wherein step (e) is performed by
 - (e1) calculating a covariance matrix Σ from a solution of the filter algebraic Riccati equation in the form
$$\begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix} \Sigma + \Sigma \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}' + \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix} \Xi \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix}' - \Sigma \begin{pmatrix} 0 & I \end{pmatrix}' \Theta^{-1} \begin{pmatrix} 0 & I \end{pmatrix} \Sigma = 0,$$
where M is the inertia matrix, Θ is the internal disturbance matrix and Ξ is the external disturbance matrix, and
 - (e2) calculating the Kalman filter gain H from $H = \Sigma \begin{pmatrix} I & 0 \end{pmatrix}' \Theta^{-1}$.

8. (original) The process of claim 7, wherein step (f) is performed by

f1) solving $H = \begin{pmatrix} M^{-1}B \\ M^{-1}K \end{pmatrix}$ for B and K, and

f2) setting the optimal mount damping matrix to B and the optimal mount stiffness matrix to K.

Appendix B: Cited References

- Exhibit A - Kim et al., H speed control of an induction motor with the two-mass resonant system by LMI, IEEE IECON '98, pp. 1439-1444, August 1998.
- Exhibit B - Brown et al., Development of a servowriter for magnetic disk storage applications, IEEE Trans. Inst. & Meas., pp. 409-415, April 1990.
- Exhibit C - Lee, The Hinf model matching approach to motor tracking control system, IEEE Conf. Indust. Automation & Control, pp. 624-631, May 1995.
- Exhibit D - Reif et al., Nonlinear state observation using H filtering Riccati design, IEEE Trans. Automatic Control, pp. 203-208, January 1999.
- Exhibit E - Shah et al., U.S. Patent No. 5,991,525, Filed August 22, 1997.

Appendix C: Evidence Appendix

none

Appendix D: Related Proceedings Appendix

There are no known related appeals or interferences regarding the present appeal.



IN THE UNITED STATES PATENT AND TRADEMARK OFFICE
BEFORE THE BOARD OF PATENT APPEALS AND INTERFERENCES

First Named
Inventor : Pradeep Kumar Subrahmanyam

Appln. No. : 09/651,046

Filed : August 30, 2000

For : PROCESS FOR DESIGNING AN OPTIMAL
VIBRATION ISOLATION MOUNT FOR A
DISC DRIVE

Docket No. : S01.12-0632/STL 9334.00

Appeal No.:

Group Art Unit: 2128

Examiner: H. M. Jones

**TRANSMITTAL OF APPEAL BRIEF
(PATENT APPLICATION - 37 C.F.R. §41.37)**

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18th DAY OF July, 2005

PATENT ATTORNEY

Sir:

Transmitted herewith is the Appeal Brief in this application with respect to the Notice of Appeal filed on June 13, 2005.

FEE FOR FILING APPEAL BRIEF

Pursuant to 37 C.F.R. §41.20(b)(2) the fee for filing the Appeal Brief is \$500.00.

The Director is authorized to charge any additional fees associated with this paper or credit any overpayment to Deposit Account No. 23-1123.

Respectfully submitted,

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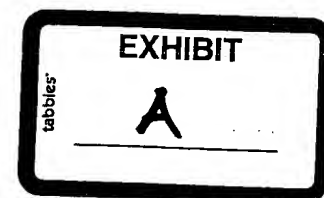
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H_∞ speed control of an induction motor with the two-mass resonant system by LMI

Jin-Soo Kim Young-Seok Kim Jae-Hwa Shin Hyun-Jung Kim

Dept. of Electr. Eng., Inha Univ., Incheon, South Korea;

This paper appears in: Industrial Electronics Society, 1998. IECON '98. Proceedings of the 24th Annual Conference of the IEEE

Meeting Date: 08/31/1998 - 09/04/1998

Publication Date: 31 Aug.-4 Sept. 1998

Location: Aachen Germany

On page(s): 1439 - 1444 vol.3

Volume: 3

Reference Cited: 10

Number of Pages: 4 vol. xxix+2635

Inspec Accession Number: 6142330

Abstract:

In industrial motor drive systems, a shaft torsional **vibration** is often generated when a motor and a load are connected with a flexible shaft. This paper treats the **vibration** suppression control of such a system. In this paper, a control system design method using linear matrix inequality (LMI), which is a tool for control design that replaces or complements Lyapunov-**Riccati** equations, is provided and the H_∞ speed controller for an induction motor by LMI is proposed. In the H_∞ speed controller, weights are used to satisfy tracking and disturbance rejection. Experimental results show the validity of the proposed H_∞ speed controller by LMI, and this controller is compared with the state feedback controller

Index Terms:

H_∞ control control system synthesis induction motor drives machine control machine testing machine theory variable speed drives velocity control **vibration** control H_∞ speed control Lyapunov-**Riccati** equations control design control performance disturbance rejection induction motor industrial motor drive system linear matrix inequality shaft torsional **vibration** state feedback controller tracking two-mass resonant system **vibration** suppression control

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H_∞ Speed Control of an Induction Motor with the Two-Mass Resonant System by LMI

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Abstract - In the industrial motor drive system, a shaft torsional vibration is often generated when a motor and a load are connected with a flexible shaft. This paper treats the vibration suppression control of such a system. In this paper, a control system design method using Linear Matrix Inequality (LMI), which is a tool for control design that replaces or complements Lyapunov-Riccati equations, is provided and the H_∞ speed controller for an induction motor by LMI is proposed. In the H_∞ speed controller, weights are used to satisfy tracking and disturbance rejection. Experimental results show the validity of the proposed H_∞ speed controller by LMI, and this controller is compared with the state feedback controller.

1. Introduction

In the industrial motor drive systems such as industrial robots, rolling mills, elevators and so on, the shaft torsional vibration is often generated when a motor and a load are connected with a flexible shaft. Such a system is called as the two-mass resonant system and this paper treats the vibration suppression control of this system.

To suppress the shaft torsional vibration, the resonant ratio control, the state feedback control, the LQG control and the H_∞ control are studied.^[1-6] It is often impossible to detect the load speed, the shaft torsional torque and the load disturbance torque in the real systems. Recently, it is studied the new method which estimates unknown state variables by using the reduced order observer and feed back these state variables by using the pole placement design method.^[7] But this design method doesn't satisfy the fast speed response property and

the attenuation property of load disturbance at the same time.

In recent studies, there is the H_∞ control which satisfies the fast speed response property and the attenuation property of load disturbance at the same time by using the weights. The H_∞ control is often applied in the industrial systems.

In recent, because it is possible to represent the control specifications and the robust stable conditions by Linear Matrix Inequality (LMI), the control problem can be transformed into the convex problem.^[8,9] LMIs have gained much attention as a tool for control design that replaces or complements Lyapunov-Riccati equations.

In this paper, the controller design method by LMI is provided and the H_∞ speed controller of an induction motor by LMI is proposed. The validity of the proposed controller is confirmed by experimental results and this controller is compared with the conventional state feedback controller using the pole placement method.^[7]

2. The two-mass resonant system

The two-mass resonant system which has a motor and a load connected with a flexible shaft is shown in fig. 1. The state equation of the two-mass resonant system is as follows

$$\begin{bmatrix} \dot{\omega}_M \\ \dot{\omega}_L \\ \dot{T}_{SH} \end{bmatrix} = \begin{bmatrix} -\frac{B_M}{J_M} & 0 & -\frac{1}{J_M} \\ 0 & -\frac{B_L}{J_L} & \frac{1}{J_L} \\ K_{SH} & -K_{SH} & 0 \end{bmatrix} \begin{bmatrix} \omega_M \\ \omega_L \\ T_{SH} \end{bmatrix}$$

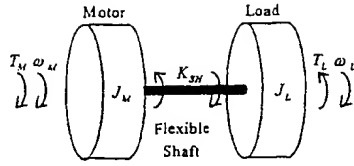


Fig.1 The plant model of the two-mass resonant system

$$+ \begin{bmatrix} \frac{1}{J_M} & 0 \\ 0 & -\frac{1}{J_L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_M \\ T_L \end{bmatrix} \quad (1)$$

where,

J_M, J_L : the motor inertia, the load inertia

B_M, B_L : the motor viscous damping coefficient, the load viscous damping coefficient

K_{SH} : the shaft stiffness

ω_M, ω_L : the motor speed, the load speed

T_M, T_{SH}, T_L : the motor torque, the shaft torsional torque, the load disturbance torque

The goals for the control of the two-mass resonant system are stated in the following forms.

- (1) Suppress the shaft torsional vibration.
- (2) Reject the effect of the load disturbance torque.
- (3) For the change of the speed reference, the load speed follows it quickly without overshoot.

3. The H_∞ generalized plant

Fig. 2 is the H_∞ generalized plant to apply the H_∞ control theory to the two-mass system. In fig. 2, w_1, w_2 are inputs for disturbance and command, respectively. and z_1, z_u are outputs. z_1 is as follows.

$$z_1 = C_z x = \begin{bmatrix} \omega_M \\ \omega_M - \omega_L \\ \omega_L \end{bmatrix}, C_z = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

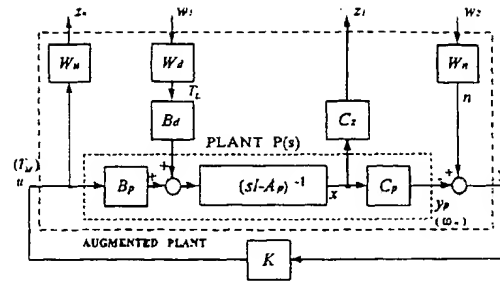


Fig. 2 The generalized plant of the H_∞ control

T_L is the disturbance torque and n is the speed command. W_d is weight for T_L and W_n is weight for n . If weights are defined as follows,

$$W_d = \begin{bmatrix} A_{wd} & B_{wd} \\ C_{wd} & D_{wd} \end{bmatrix}, W_n = \begin{bmatrix} A_{wn} & B_{wn} \\ C_{wn} & D_{wn} \end{bmatrix}, W_u = D_{wu} \quad (3)$$

then the state space equation for the generalized plant and the ratio of output to input, G , are as follows.

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix} \quad (4)$$

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} = \begin{bmatrix} A_p & 0 & B_p C_{nw} & 0 & B_p D_{nw} & B_p \\ 0 & A_{nw} & 0 & B_{nw} & 0 & 0 \\ 0 & 0 & A_{pw} & 0 & B_{pw} & 0 \\ C_x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{uw} \\ -C_p C_{nw} & 0 & D_{nw} & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

Above G is obtained by choice of W_d, W_n, W_u and the H_∞ controller K is solved by Matlab.

4. The control system design by LMI

In this chapter, we represent G of the generalized plant by LMI. Convexity of LMI and LMI representation of Riccati inequality for the control design are given and convex problems involving LMI are treated.

Riccati inequality applying to control system design is as follows.

$$\Leftrightarrow \gamma^2 V - F^T V F > 0; \quad V := W^T W > 0 \quad (10)$$

where W : scaling matrix, γ : level

$$A^T S + SA + SBR^{-1}B^T S + Q < 0, \quad R > 0 \quad (6)$$

In Eq. (6), A and B are

$$A = \begin{bmatrix} A_p & 0 & B_d C_{wd} \\ 0 & A_{wh} & 0 \\ 0 & 0 & A_{wd} \end{bmatrix}, \quad B = [B_1 \ B_2] = \begin{bmatrix} 0 & B_d D_{wd} & B_p \\ B_{wh} & 0 & 0 \\ 0 & B_{wd} & 0 \end{bmatrix} \quad (7)$$

in real two-mass system and Q, R, S can be selected by free choice. Eq. (6) can be transformed into

$$\begin{bmatrix} -A^T S - SA - Q & SB \\ B^T S & R \end{bmatrix} = F(e) > 0 \quad (8)$$

by using Schur complement₍₁₀₎ And Eq. (8) can be converted to following general form of LMI.

$$F(e) = F_0 + \sum_{i=1}^m e_i F_i \quad (9)$$

where

$$F_0 = \begin{bmatrix} -Q & 0 \\ 0 & R \end{bmatrix}, \quad F_i = \begin{bmatrix} -A^T S_i - S_i A & S_i B \\ B^T S_i & 0 \end{bmatrix},$$

e_i : vector

Convex problems for solving $F(e) > 0$ are remarkably two problems as follows.

1) Convex Feasibility Problem (CFP)

find $e \in \mathcal{R}^m$ such that $F(e) > 0$

2) Convex Minimization Problem (CMP)

minimize $c^T e$ subject to $F(e) > 0, \quad e \in \mathcal{R}^m$

where $c \in \mathcal{R}^m$ is given.

And because of scaled matrix norm condition, $\|WFW^{-1}\| < \gamma$, can be transformed into eq. (10),

$$\begin{aligned} \|WFW^{-1}\| < \gamma &\Leftrightarrow W^{-T} F^T W^T W^T F W^{-1} < \gamma^2 I \\ &\Leftrightarrow F^T W^T W F < \gamma^2 W^T W \end{aligned}$$

following problems can be obtained.

1) CFP : when matrix F and level $\gamma > 0$ are given, problem solving scaling matrix W (or V);

$\exists V := W^T W > 0$ such that $\gamma^2 V - F^T V F > 0$

2) CMP : when matrix F and scaling matrix W are given, problem minimizing level γ ;

minimize $\gamma > 0$ such that $\gamma^2 V - F^T V F > 0$

We can get W (or V) in case of minimum γ by iteration.

5. H_∞ speed control of the two-mass system by LMI

We design the H_∞ controller of the two-mass system by LMI. In fig. 3, the weighting function W_u is selected as $W_u = 10^{-6}$ which does not influence the design of controller. The choice of weighting functions is very critical because the control performances are determined by weighting functions. The weighting function W_d is for the closed-loop transfer function from the disturbance torque T_L to z_1 . In order to achieve the design specifications such as vibration suppression and disturbance rejection, $|W_d|^{-1}$ is chosen as closed-loop gain plot of the one-mass system. The weighting function W_n is for the closed-loop transfer function from the disturbance torque n to z_1 , where n corresponds to the speed command or noise. The closed-loop transfer function from n to ω_M is a complementary sensitivity function, so W_n is chosen as a high pass filter considering the robust stability. Then W_d and W_n are chosen as follows.

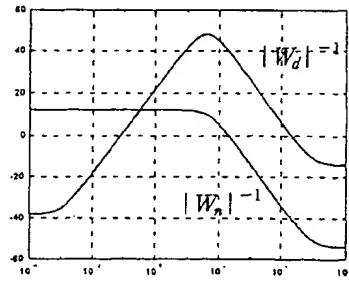


Fig. 3 Bode diagram of weights
(ver : [dB], hor : [ω])

$$W_d = \gamma_d \frac{(s + \omega_d)}{(s + 10^{-3})(s + 10^5)}$$

$$W_n = \gamma_n \frac{(s + 50)}{(s + 10^5)} \quad (11)$$

The H_∞ controller $K(s)$ can be obtained by using Matlab after adjusting the parameters in weighting functions as $\gamma_d = 5$, $\omega_d = 40$, $\gamma_n = 500$. Weighting functions are shown in fig. 3. The H_∞ controller $K(s)$ is given by

$$K(s) = 4.921 \times 10^4 \times \frac{(s + 1 \times 10^3)(s + 0.6 \pm j80)(s + 5.41)}{(s + 5.6 \times 10^5)(s + 3.59 \times 10^4)(s + 9 \pm j33.52)(s + 0.0011)} \quad (12)$$

6. Experimental system

The configuration of the experimental system for the two-mass resonant system is shown in fig. 4. There is a flexible shaft (l : 800 [mm], ϕ : 13 [mm]) between a motor and a load generator. This system is used variable inertia to change the inertia of a motor or a load generator. For this paper, it is only used variable inertia for a load generator. The mechanical constants of the two-mass resonant system are listed on Table 1.

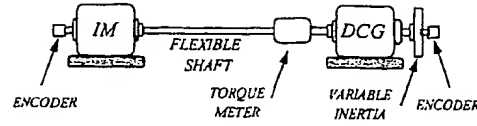


Fig. 4 The experimental system for the two-mass resonant system

Table 1 The mechanical constants of the two-mass resonant system

J_M	J_L	K_{SH}
0.008 [kgm ²]	0.08 [kgm ²]	50.527 [Nm/rad]

7. Experimental results

Experimental results show the validity of the proposed controller and this controller is compared with the state feedback controller. The inertia ratio is 0.008 : 0.08 (J_M : J_L) in this paper. Experiments are composed of speed response experiment and load disturbance response experiment.

7.1. Speed response experiment

The reference speed of the load is 200[rpm] as a step input at 0 [sec]. Fig. 5, 6 are experimental results of step response under the state feedback control. Fig. 5 shows load speed and motor speed and fig. 6 shows shaft torsional torque. The speed transient time from 0 to 200 [rpm] is about 0.7 [sec]. If the transient time becomes shorter, overshoot of the load speed is larger. In the transient time, maximum torque is 6 [Nm].

Fig. 7, 8 are experimental results for control by LMI. Because the speed transient time from 0 to 200 [rpm] is about 0.5 [sec], the transient time of proposed LMI controller has much less than that of state feedback controller. This is because of adjusting the weighting function for speed response. In the transient time, maximum torque is 9 [Nm]. But this maximum torque does not influence the two-mass system.

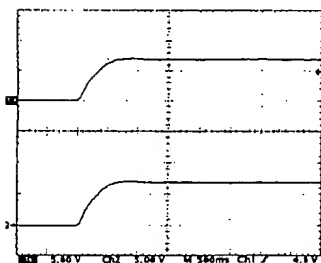


Fig. 5 State feedback controller
150 [rpm]/div, 0.5 sec/div
(upper) Real motor speed, (down) Real load speed

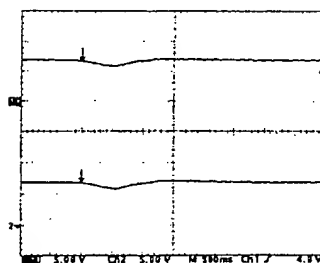


Fig. 9 State feedback controller
150 [rpm]/div, 0.5 sec/div
(upper) Real motor speed, (down) Real load speed

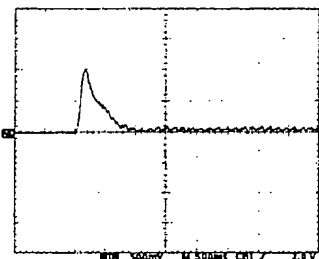


Fig. 6 State feedback controller
3 [Nm]/div, 0.5 sec/div
Real shaft torsional torque

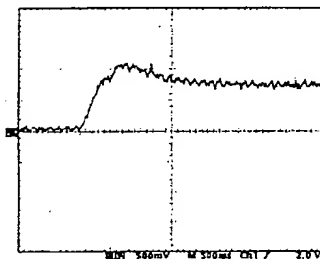


Fig. 10 State feedback controller
3 [Nm]/div, 0.5 sec/div
Real shaft torsional torque

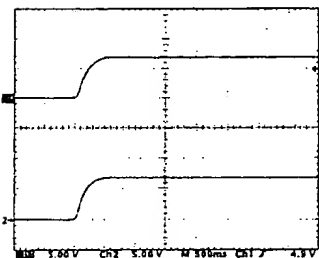


Fig. 7 LMI controller
150 [rpm]/div, 0.5 sec/div
(upper) Real motor speed, (down) Real load speed

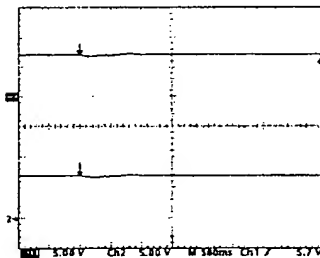


Fig. 11 LMI controller
150 [rpm]/div, 0.5 sec/div
(upper) Real motor speed, (down) Real load speed

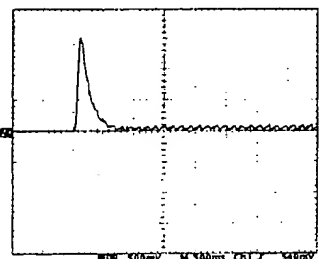


Fig. 8 LMI controller
3 [Nm]/div, 0.5 sec/div
Real shaft torsional torque

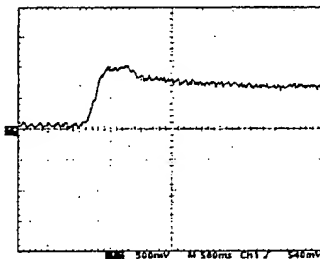


Fig. 12 LMI controller
3 [Nm]/div, 0.5 sec/div
Real shaft torsional torque

7.2. Load disturbance response experiment

A resistor load which can be connected with DC generator is used as a load. By using a resistor load, the load disturbance torque is increased from 0 to 4.5 [Nm] at 200 [rpm]. 4.5 [Nm] is 37% of rated torque. Fig. 9, 10 are experimental results for the state feedback control. Load speed dropped 17% of 200 [rpm] and recovered after 1.2 [sec]. This is because the inertia ratio is big. If the inertia ratio increases, the recovery time is much longer.

Fig. 11, 12 are experimental results for control by LMI. The load speed dropped 5% of 200 [rpm] and recovered after 0.6 [sec]. Then, the speed drop of the proposed LMI controller has much less than that of state feedback controller. The recovery time of the proposed LMI controller has much less than that of the state feedback controller.

8. Conclusion

This paper treated the speed control of the induction motor system which has a motor and a load connected with a flexible shaft. The H_∞ speed control of an induction motor with two-mass resonant system by LMI is proposed. In the proposed control, the H_∞ control, which satisfies the fast speed response property and the attenuation property of load disturbance at the same time by using the weights, is used and is represented by LMI which is a tool for control design that replaces or complements Lyapunov-Riccati equations.

The experiments show the excellent performance of the proposed H_∞ speed control by LMI compared with the state feedback control.

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Title	"Optimum Vibration Control of Distributed Parameter Rotor Bearing System by Using Electromagnetic Bearing"
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Abstract

As the modern rotor systems have a tendency of being operated at high speed and designed to be lighter, the control of excessive vibration becomes more and more important because of increasing requirement in accuracy and reliability. Since the flexible rotor bearing system is a typical hyperbolic type distributed parameter system, the control, passive or active, not accounting for such a nature often does not yield satisfactory performance. The primary objective of this study is to develop efficient control design methods which account for and use the distributed parameter and isotropic properties. The output feedback control scheme accounts for the practical aspects such as the observation and related stability problems caused by spillover effects, while the optimal design deals mainly with the theoretical and numerical aspects. Optimal pole assignment in distributed parameter control systems is established and the existence of optimal solution is proven in case of compact linear feedback. The method, an extension of the finite dimensional design, gives the solution of the operator Riccati equation which is known to be one of the most difficult problems faced by the designers. By adopting the complex notation for the isotropic systems, it is shown that the closed loop eigenstructure is determined, contrary to the real field design, so as to preserve the isotropic property. In order to utilize the symmetric property between the two directional vibrations: vertical and horizontal or forward and backward, a new control scheme is designed by introducing the complex state for the systems having the isotropic property. The algebraic relations concerning distribution of the eigenvalues of complex matrices are developed so as to cluster the poles into specified regions, rather than fixed positions, imposing the relative stability margin, e.g., uniform damping or uniform damping ratio. The closed loop eigenstructure has the direct one-to-one relation with the physical parameters and the isotropic property is preserved under feedback control. The method transforms Riccati equation to Lyapunov equation of the same dimension so that computational effort is almost removed. It is shown that the method can be used effectively in vibration control of rotating machinery such as the isotropic rotor bearing system, the magnetic bearing system and the rotating circular disk. To achieve the dual objective of stabilization and spillover suppression, a constrained output feedback control scheme is developed based on the optimization of a modified performance index which includes spillover terms. In order to construct the appropriate performance index, the method makes use of the two-time scale property of the primary and secondary modes. The control model is constructed based on the singularly perturbed modal model, which retains the simple modal structure while preserving the accuracy of the frequency domain modeling. A set of optimality condition is derived and iterative solution procedure is also presented. A set of simulation examples show the effectiveness of the proposed scheme. In order to control the rotation related periodic disturbances, a disturbance accommodating controller is also designed based on the disturbance estimator. The main advantage of the proposed scheme is the simplicity in numerical calculation and estimator structures, requiring only a solution of non-symmetric Lyapunov equation of relatively small order. Lyapunov equation is the first order perturbation solution of the matrix cubic equation which yields the exact solution. Through the matrix perturbation analysis, it is proven that, for small gain, the approximate solution converges to the exact one. The control scheme is proven to effectively control the build up vibration subject to sudden imbalance through the numerical simulation. Finally, the output feedback control experiment is performed by using a magnetic bearing when the rotor system is subject to external impulse and imbalance. The magnetic bearing with four magnets is manufactured and the analog design is transformed to the discretized form and implemented by using a 16 bit microcomputer. The control model is constructed by using the experimentally obtained modal parameters through a set of modal testings in complex coordinates. The proposed control scheme enables the rotor to run through the first critical speed without excessive vibration against the initial system unbalance. In order to verify the performance of the proposed control



initial system unbalance. In order to verify the performance of the proposed control scheme, the results are compared with the numerical ones. It is shown that the experimental results are fairly well coincident with the predicted ones. In conclusion, through the extensive analysis and experiments, it is shown that the proposed controllers can be used effectively in practical implementation as well as in theoretical and numerical aspect of controller design for the rotor bearing systems.

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Development of a servowriter for magnetic disk storage applications

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Abstract:

The design, development, and application of a research servowriter, which is necessary to record magnetic position encoding schemes, are presented. The servowriter is designed to evaluate emerging disk drive technologies. Submicron positioning accuracy of the disk actuator is achieved by integrating the **state**-of-the-art electronic, optic, electromechanical, and digital signal processing hardware. The servowriter also provides flexible and sophisticated signal encoding schemes through a programmable pattern generator and has several modules of customized electronic hardware, each controlled by a supervisory computer. The position of the R/W (read/write) head and actuator assembly is measured by a laser interferometric system. The measured position is used to estimate the velocity of the R/W head, which is then used to implement a **state**-variable-based proportional-integral-derivative controller on the Intel (8086/8087) microprocessor chip set. Both systematic and random error sources associated with phase and amplitude encoding schemes are described. Environmental factors such as seismic **vibration** and temperature change, which indirectly contribute to servowriting errors, are identified, and steps to contain these error sources are described.

Index Terms:

computerised control encoding laser beam applications light interferometry magnetic disc storage microcomputer applications position control servomechanisms three-term control velocity control Intel amplitude encoding disk drive error laser interferometric system magnetic disk storage magnetic position encoding microprocessor chip modules phase encoding

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Development of a Servowriter for Magnetic Disk Storage Applications

DANA H. BROWN AND MUTHUTHAMBY SRI-JAYANTHA, MEMBER, IEEE

Abstract—The design, development, and application of a research servowriter, which is necessary to record magnetic position encoding schemes, are presented. The servowriter is designed to evaluate emerging disk drive technologies. Submicron positioning accuracy of the disk actuator is achieved by integrating the state-of-the-art electronic, optic, electromechanical, and digital signal processing hardware. The servowriter also provides flexible and sophisticated signal encoding schemes through a programmable "pattern generator" and has several modules of customized electronic hardware, each controlled by a supervisory computer. The position of the R/W (read/write) head and actuator assembly is measured by a laser interferometric system. The measured position is used to estimate the velocity of the R/W head, which is then used to implement a state-variable-based proportional-integral-derivative controller on the Intel (8086/8087) microprocessor chip set. Both systematic and random error sources associated with phase and amplitude encoding schemes are described. Environmental factors, such as seismic vibration and temperature change, which indirectly contribute to servowriting errors, are identified and steps to contain these error sources are described.

I. INTRODUCTION

THE HIGH-performance disk drives of today are precision rotating machines, with head positioning mechanisms capable of achieving submicron track following accuracy. As the demand for track density increases, the ability to provide a higher resolution position signal will become mandatory. In order to maintain the magnetic R/W (read/write) head position along the center line of a desired track, a magnetic pattern that indicates the position of the R/W head is prerecorded on the disk surfaces during the manufacture of the disk drive. The device used to record the magnetic pattern is called a servowriter [1, p. 719]. Since the magnetic pattern helps the servo control algorithms perform the track-following or track-accessing task, it is also referred to as a servo pattern.

Of the several major steps involved in the manufacture of a disk file, the servowriting operation takes place just prior to sealing the head/disk assembly in a clean room environment. A servowriter in itself is a complex device that integrates state-of-the-art electronic, optic, and electromechanical hardware with sophisticated signal encoding schemes to achieve the submicron accuracies that are

demand in the industry. There are three broad categories of position encoding techniques used by the disk drive industry:

- 1) nonmagnetic position encoding (optical sensors mounted on the head actuator, mechanical detents for each track, stepper motors, etc);
- 2) magnetic servo pattern based position encoding (amplitude encoded schemes [2] and phase encoded schemes [3]);
- 3) combinations of the above (optical sensors with a single servo burst per track to remove low frequency track-following errors).

The servo control of the IBM9332, for instance, is based on a phase encoded servo pattern [4], whereas the IBM33XX series is based on an amplitude encoded servo pattern. A short overview of servo patterns is given in the Appendix.

This paper addresses the design, development, and application of a research servowriter that is targeted to achieve the magnetically based position encoding technique in category (2). Following an overview of the servowriter, its major elements are described briefly. The servowriter electronics are made up of several customized electronic hardware modules, including an Intel (8086/8087) microprocessor chip set. Each module or subsystem is associated with a function that is required to support the servowriting and testing process and is controlled by a supervisory computer (IBM-PC/AT). The pattern generator is software configurable and is controlled by the IBM-PC/AT. The IBM-PC/AT is menu driven and it is through this menu that the user controls or monitors the servowriting operation. More details are presented in [5].

The position of the R/W head is measured by a laser interferometric system [6]. The laser based position is used to estimate the state variables of the R/W head positioning mechanism. These estimates are then used to implement a state-variable-based proportional-integral-derivative (PID) controller, which positions the mechanism with zero steady-state error in the presence of bias force. Estimator/controller algorithms are implemented in 8087-based floating-point arithmetic using assembly language. An example of a closed-loop transfer function of a servowriting mechanism is presented, and factors that would contribute to track-following error, which is detrimental to the servowriter performance, are discussed.

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Details related to clock pulse generation that is mandatory for locating the pattern generator pulses are described. The servo pattern is generated from 1024×32 bit memory-based data, thus providing flexibility to study the performance of various encoding schemes. Finally, the error sources associated with encoding schemes, and environmental factors such as seismic vibration and humidity, which indirectly contribute to servowriting errors, are identified and steps to contain these error sources are described.

II. SERVOWRITER DESCRIPTION

The servo patterns are written on one or more disk surfaces, depending on the design of the file being written (i.e., dedicated versus sector servo). The servo patterns can be written with heads mounted on the disk drive actuator, or a special set of heads can be mounted to the servowriter to perform this task. The servowriting process is a repetitive process of writing a track and moving the head to the next track. The process repeats itself until all tracks are written. The process of writing the servo patterns is relatively straightforward, but since it must be precisely done, the circuitry to do this can become quite complex. Fig. 1 shows the major elements of a servowriter, some of which are briefly described now.

The heads must have a sensor mounted to them to indicate their radial position on the disk. Typically a laser interferometer is used to measure the head position. This sensor system is used in a closed-loop servo system to locate each track to be servowritten. Once the head is positioned over a track, the pattern is written. However, the pattern must be synchronized to the disk rotational position so that adjacent concentric tracks line up according to a selected encoding rule. For this task, a sensor is used to supply fine resolution clock pulses that are synchronized to the angular position of the disk. This sensor is typically a stationary R/W head called a clock head. As the spindle motor speed varies due to random disturbances, the clock pulses follow the speed changes accordingly.

An encoder or pattern generator supplies the correct servo code to the heads for writing on the disk. This pattern generator follows the angular position clock as it writes each track so that the servo pattern on that track is aligned to the servo pattern that was written on the previous adjacent track. The pattern generator is programmed to change the code written on each track so that the servo signal on read back will indicate over which track the head is positioned.

Each subsystem, together with the tasks performed by it, is coordinated by a central supervisory computer. A suitable architecture is used to share the computing, control, and monitoring tasks required by a servowriter.

III. MAJOR ELEMENTS OF A SERVOWRITER

A. Computer Architecture

Fig. 2 shows the microprocessor hardware interconnection necessary to achieve the servowriting operation. The

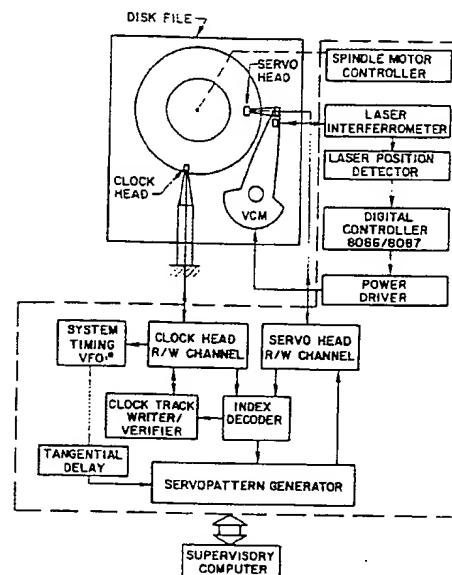


Fig. 1. Elements of a servowriter (* VFO: variable frequency oscillator).

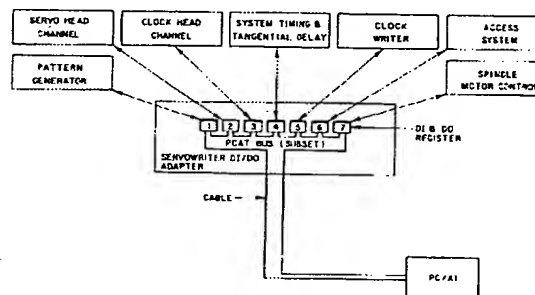


Fig. 2. Computer architecture.

servowriter is controlled by an IBM PC/AT with external data output (DO) and data input (DI) registers. The control program for this servowriter was written in IBM interpreter BASIC with some data manipulation programs (such as the pattern generator down-loader) written in 8088 Assembly language. The memory in the PC/AT is 640 kilobytes. However, the program to control the servowriter is contained in a 64 kilobyte segment. Although interpreter BASIC is slow compared to Assembly language, 2000 servo half tracks can be written on a disk in well under 1 h. This is considered adequate for a research servowriter, but is not acceptable for a production servowriter. The software would have to be optimized to reduce that time by a factor of at least 5.

B. Digital Position Control System

The head is positioned by mounting a retroreflector to the actuator and tracking it using laser interferometry. A closed-loop digital servo feedback system is employed to hold the head position to within a few microinches of the

target position. A second-order actuator model with an augmented position integral state is used for controller design. A moving coil actuator with a current-mode driver is thus modeled as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ -k_s/m & 0 & -c_s/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ k_F/m \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} x_{Ref} + w \quad (1)$$

$$z = [k_i, 0 \ 0 \ 0]x + v \quad (2)$$

where

- c_s damping constant ($N \cdot s/m$),
- k_F actuator force constant (N/A),
- k_s stiffness constant (N/m),
- k_i sensor gain constant (laser counts/ m),
- m equivalent actuator mass (kg),
- u control current commanded by servowriter processor (A),
- v laser measurement noise (equivalent laser count),
- w process noise vector (appropriate units),
- x^T [position (m), integral ($m \cdot s$), velocity (m/s)],
- x_{Ref} reference position (m),
- z measured position (laser counts).

The representative model defined by (1) is used to design a PID-type controller using a pole-placement design technique. The introduction of process and measurement noise terms emphasizes that the system is not deterministic, and that the tracking performance, therefore, can never be ideal (i.e., zero error). The unmodeled disturbances act through the process noise term, w , and measurement noise appears through the noise term, v . Seismic (or ground) vibration not only affects the accuracy of the model defined by (1), but also contributes to measurement noise by causing the laser optics to move relative to the actuator. Isolation of these sources contributes significantly to the cost of a servowriter.

The continuous model is discretized for controller/estimator design. A sampling rate of $300 \mu s$ is used to discretize the model and is represented as follows (ignoring noise terms, and setting $x_{Ref} = 0$):

$$x_{k+1} = \Phi x_k + \Gamma u_k \quad (3)$$

$$z_k = h^T x_k \quad (4)$$

where

- k sample index,
- h measurement matrix as defined in (2).

Equations (3) and (4) are used to select the controller feedback gain constants. A third-order Butterworth pole pattern is used to locate the desired closed-loop poles of the position servo.

Since the measured position (i.e., laser count) is not a continuous variable, a reduced order "current" estimator is designed to obtain the velocity estimate [7, p. 149]. In the design of the estimator, measured position is assumed to be exact, and the position (error) integral state is excluded from the estimator model. Therefore, the velocity estimator is of first order, and the pole-placement design yields a single gain corresponding to a desired estimator real pole location. The integral state is derived using the trapezoidal rule for integration.

The controller/estimator gains and the system matrix elements are arranged together so that each laser position measurement is used to generate an appropriate electric current command in an explicit state variable form. The controller/estimator computations are realized in floating point arithmetic. Using a Hewlett Packard 3562A dynamic signal analyzer, a closed-loop transfer function (CLTF) of an experimental system was obtained. Fig. 3 shows the measured CLTF of an integrated disk/actuator system under servowriter control. The bandwidth of this system is limited by a 400-Hz resonance mode specific to the actuator used (in which the mounting of the retroreflector accentuated the 400-Hz mode). The servowriter controller/estimator algorithm has adequate computational speed to support a closed-loop system with much higher bandwidth if the actuator modes are not a problem from a stability standpoint. The presence of the integral state provides the required low-frequency disturbance rejection. In order to suppress high-frequency disturbance, greater bandwidth with minimum CLTF peaking is recommended.

C. System Timing and Clock Writer Design

There are different methods in practice today for writing clock signals. The simplest of all methods is to let the clock writer run freely at a designed clock frequency and to dither the disk speed (because it can never be regulated perfectly at a constant speed) until *closure* (i.e., an integral number of complete clock cycles around a track) is achieved. This task is automatically performed by the clock writer in 5 to 30 s. Hence, as defined in the next paragraph, the total number of clock cycles is first selected for a given problem, and through repeated write trials, the clock signal is recorded until satisfactory closure is met.

The total number of clock cycles per track is based on four parameters that have various system implications. They are defined as follows:

- Δ servo pattern sampling period (s),
- N_{rpm} spindle motor speed (rpm),
- f_{clock} clock track frequency (MHz),
- $m_{section}$ integer number of sectors per revolution.

The sampling time is specified to meet the performance requirements of the disk actuator servo. (Note that the servowriter laser sampling rate is independent of the disk servo sampling rate.) The spindle speed is usually determined from magnetic considerations and is set to a spec-

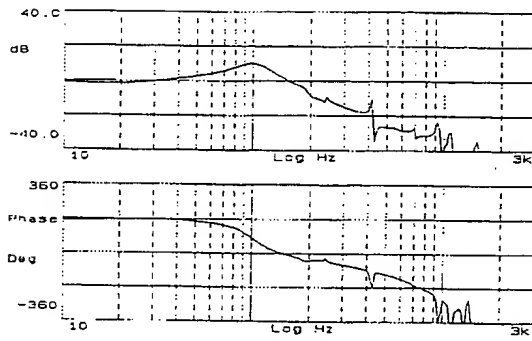


Fig. 3. Closed loop transfer function of an experimental system.

ified value. The clock frequency should meet the R/W head frequency characteristics as well as the requirements imposed by the specific servo encoding scheme. The number of sectors is, however, constrained by the relationship

$$m_{\text{sector}} = \frac{1}{\Delta} \times \frac{60}{N_{\text{rpm}}} \quad (5)$$

The initial estimate of m_{sector} will not necessarily be an integer. Therefore, a convenient integral number of sectors, m_{sector}^* , corresponding to an exact, but slightly adjusted, sampling rate Δ^* , is selected as the first step. For instance, in order to drive the brushless dc spindle motor, an even number of sectors divisible by 12 (related to the number of motor poles) may be necessary. Based on the design requirements of the servo pattern, n_{sector} clock cycles at a frequency f_{clock} are to be generated between two consecutive sectors with a duration dictated by an adjusted sampling rate Δ^* . This requirement is met by the relationship

$$n_{\text{sector}} = \Delta^* \times f_{\text{clock}} \quad (6)$$

In the case where n_{sector} of (6) does not result in an integer, it may be rounded to the nearest integral value, n_{sector}^* . Fractional adjustment of clock frequency (in megahertz) would typically satisfy this constraint. Hence, the total number of clock cycles per revolution of the disk spindle is

$$n_{\text{total}} = n_{\text{sector}}^* \times m_{\text{sector}}^* \quad (7)$$

This redefines the spindle speed to be

$$N_{\text{rpm}}^* = f_{\text{clock}} \times \frac{60}{n_{\text{sector}}^* \times m_{\text{sector}}^*} \quad (8)$$

which, though not exact, is near the desired spindle speed. For example, a consistent set of parameters would be $\Delta^* = 99.206 \mu\text{s}$, $n_{\text{sector}}^* = 248 [= 99.206 \times 10^{-6} \times (2.5 \times 10^6)]$, $n_{\text{total}} = 248 \times 168 = 41\,664$ clock cycles per revolution, and $N_{\text{rpm}}^* = 3600.23$ rpm, where the clock track writer is programmed to write n_{total} ($= 41\,664$) clock cycles at a frequency of 2.5 MHz. This is done by turning on the write current to the clock head while writing 2.5-MHz data to it. In order to have exactly 41 664 clocks written under the clock head, a counter must register how

many cycles have been written during each revolution of the disk. When exactly 41 664 clocks have been written for one revolution (achieved by dithering the speed of the motor slightly around N_{rpm}), the write current is shut off.

Counting clock pulses during clock writing requires a reference mark to indicate precisely when the disk has turned through a single revolution. This is done by writing an index mark with the servo head before clock writing. The index decoder (see Fig. 1) monitors the read-back signal from the servo head and outputs a pulse each time the index mark is detected. This pulse (called index) initially starts the clock writer counter during clock writing and resets it after each unsuccessful pass of clock writing. Clock track verification is done by simply reading back the written clock track and counting the number of clocks in one revolution. If disk defects damage one or more clock pulses, this will be reflected in the final count. An index mark is also encoded in the clock track to indicate the start of the clock track. Three cycles of 1.25-MHz frequency marker can be embedded in the clock track without seriously affecting the 2.5-MHz clock signal. This index is used to synchronize the pattern generators at the start of each servo track.

The clock track resolution of 400 ns is too coarse for writing servo patterns. A type 2 phase-locked loop (PLL) that multiplies the clock track frequency by 16 and holds the output phase of this 40-MHz signal constant with respect to the clock track is employed. The PLL output effectively subdivides the disk into 666 624 equally spaced time slots of 25 ns each.

D. Motor Speed Control

The speed control of the disk is critical during clock writing and clock read-back. If it varies too much during either clock writing or read-back, good closure will not be attained and the PLL that generates the system time base will go out of lock. This will make the writing of good servo patterns impossible. The speed of the disk is controlled by comparing motor Hall sensors (or, more accurately, the clock track signal) to a stable reference clock. For a research servowriter, the reference is derived from a high-precision (six to seven significant digits) frequency source. This is necessary for changing the number of clock cycles by a couple of counts per revolution. For 5.25- or 3.5-in disk drives, the motors are small enough in power consumption that single IC motor controllers are readily available, and with the proper compensation, they can control the motor speed to about 0.01 percent of reference speed.

E. Pattern Generator

The circuitry that writes the servo patterns on the tracks is called a pattern generator. It is usually programmable, but for servowriters with simple servo patterns it can be hardwired to write a limited number of servo codes. The pattern generator is a clock track counter that begins counting the 40-MHz PLL output clocks when the clock track index is received. At certain preprogrammed counts, the output of the pattern generator changes states to re-

verse the write current in the servo head. It is this sequence of transitions that makes up the servo code.

There are pattern generators of differing designs on the commercial test equipment market today. The pattern generator designed for a typical IBM servowriter is similar to some of the commercial ones except for instruction modifications to facilitate writing servo patterns. The output of the pattern generator is a shift register that clocks out the data with each 25-ns clock. The shift register can be loaded with 9-16 bits with each pattern generator instruction and then shifted out at the system PLL clock rate (40 MHz).

The pattern generator contains a block of memory, for example, 1024×32 bits. Each 32 bit word contains 3 bits of instruction op code, 3 bits to indicate the shift register data length, 16 bits of data, and 10 bits of op code modifier (i.e., subroutine call address, branch addresses, or loop count information). Although the instructions are simple, they allow the writing of any servo pattern used to date and, in most cases, all tracks for an entire disk can easily be contained in the 1024 locations.

F. Pattern Testing

Writing servo patterns on a disk at exactly the correct locations does not guarantee a correctly written disk. It is possible that defects on the surface of the disk were present and that the servo information was not recorded at those points. The recording head may have had a shorted turn or other defect that prevented the servo pattern from being written on the disk also.

The pattern test circuitry allows a read-back check to verify that a good servo pattern has been written. While the concepts and circuits for pattern testing are not difficult, this part of the servowriter is troublesome. Errors are neither black nor white. They fall into a broad range of values, many landing on the threshold that the pattern test uses to evaluate the correctness of the pattern. The result is that a particular disk defect or servo error may pass on some tests and fail on others, or it may pass on a particular servowriter and fail on another. The pattern testing spans several subsystems and the details are presented in [5].

IV. ERROR SOURCES

The servowriter errors can be either random or systematic and are the subject of the next section.

Error Mechanisms in the Servo Pattern

The phase encoded servo pattern looks at the average phase of many transitions with respect to a reference frequency (crystal). As a result, it is relatively immune to single transition defects and small amplitude variations. Ideally, the servo pattern phases should change continuously as the head position varies. Unfortunately the head gap geometry requires a discrete approximation. This approximation to a sloping phase angle is a $\delta/2$ ns discrete time shift for every $1/2$ track width of position change. Because of this discrete implementation, there can be ripple

in the position signal generated by the demodulator as the head is displaced. Selecting a finer discrete resolution does not solve the problem of ripple completely. Magnetic heads write tracks that are slightly wider than the width of the head gap. Each pass of the head while servowriting leaves a noise fringe field between tracks. This fringe field is a fixed width that is a bigger percentage of each track width as track spacing dimensions shrink. If quarter tracks are written with $\delta/4$ ns phase shifts, the linearity should get better (i.e., the ripple amplitude should decrease), but a larger percentage of the servo pattern would be made up of fringe field. Thus the improved linearity is more than negated by the noise induced.

In amplitude servo schemes, the demodulator output versus the actual head position becomes nonlinear as the head moves away from the track center line. This nonlinearity is very much dependent on the ratio of the head width to the track width. Thus the servo gain of the demodulator output (millivolts/micrometers) varies significantly with head width. Some amplitude servo patterns use only a single transition in each position sample. If such a transition is damaged, though, the position signal for that servo sample would be incorrect. Longer burst servo patterns overcome this problem to a great extent by averaging several redundant transitions (as shown in Fig. 6(b) in the Appendix). However, this redundancy cannot be carried too far because longer servo patterns begin to cause errors during high-speed seeks. They also consume the useful data space in files with sector servo architectures.

1) *Tangential Delay:* Due to the skew of the head on the rotary actuator as shown in Fig. 4, the tops of the transitions on track $N + 1$ do not line up with the bottoms of the transitions on track N as shown in Fig. 4(a). However, if the pattern written on track N is delayed slightly, they will line up as shown in Fig. 4(b). For such small adjustments, the 3-ns resolution is necessary. Besides head skew, nonradial motion of the actuator introduces a timing shift in the Y , or tangential, direction, as shown in Fig. 5. The tangential delay circuit can be programmed to a new correction factor for each radial servo track to remove these systematic errors. There is still a random delay due to the uncontrolled vibration of the clock head (δx_{clock}) and the servo head suspension (δy_{servo}). These two random components are time dependent, and are shown along the X and Y direction for simplicity. The effect of random vibration can be reduced by special treatment of the head suspensions or by selecting low jitter heads. Fig. 5 shows a generalized delay generating mechanism relating the track spacing (δr), radius (r), disk rotational velocity (ω), and slope of the actuator path (dy/dx). The relative time delay of a second track written next to an ideal first track (i.e., no head vibration) can be shown to be

Relative Delay

$$= [\tau \times \delta r] + \frac{((\delta y_{\text{servo}} - \delta x_{\text{clock}}) + \frac{dy}{dx} \delta r)}{r\omega} \quad (9)$$

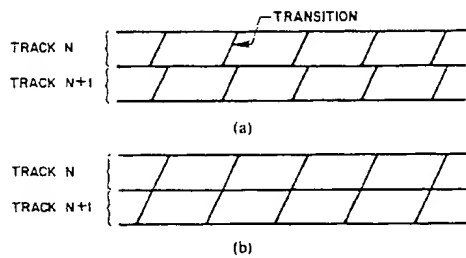


Fig. 4. Transitions with and without tangential delay correction. (The edge of a transition can either be orthogonal or inclined (as shown) to the track centerline.) (a) Without tangential delay correction the transitions do not line up track to track. (b) With tangential delay correction the transitions do line up with adjacent tracks.

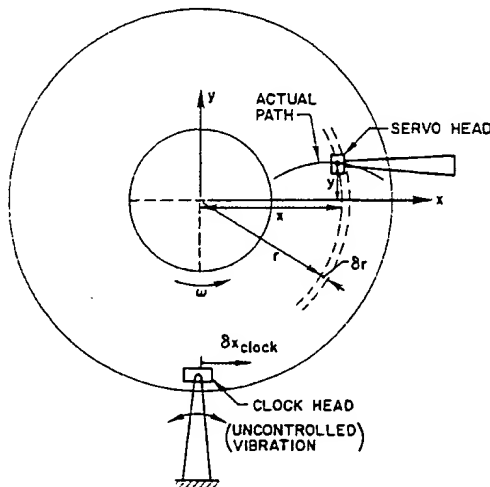


Fig. 5. Effect of nonradial head motion on tangential delay.

where τ is the ideal delay per unit distance of r . Therefore, the first group of terms in the right-hand side of (9) represents the desired ideal time delay. However, in practice the parameters defined in the second group of terms also contribute to the total delay. Observe that the delay parameters become more complicated when the actuator travels at higher velocity, as in the case of a seek.

2) *Spindle and Head Suspension Jitter*: As pointed out above, track to track timing alignment is essential. Mechanical movement of the spindle center line as it rotates will cause the disks to shift the track center line continually under the servowriter heads, even if the access system holds the heads perfectly stationary. Radial nonrepeatable spindle motion can cause the servo pattern being written to squeeze toward one of the adjacent tracks. Tangential nonrepeatable spindle motion can cause the data to be written out of alignment with the adjacent track, even if the timing signals were to occur at the proper times. Head suspension jitter can cause symptoms similar to spindle jitter. The head suspension jitter comes from both the clock head and the servowriting head. The jitter primarily occurs at right angles to the head suspension's

main axis, as the stiffness of the suspension is weakest in that direction. In-line heads reduce tangential jitter.

3) *Vibration Isolation and Mechanical Alignment*: In order to minimize file vibration during servowriting, the file is isolated from external vibrations by mounting it on a large granite base that floats on air isolation mounts. Ideally, the file should be clamped at three points, so that the base casting is not distorted while servowriting. In practice, bending modes of the casting sometimes can occur with just three clamps holding the file down and these vibrations can be written into the servo pattern in the form of track positioning errors. Thus some experimentation will usually have to be done to determine the best clamping arrangement for a particular disk drive.

The alignment of the disk and head stack for sector servowritten files is critical. During file operation, all heads on an actuator should be centered over the same track number on each surface. In addition, the servo patterns on all surfaces should pass under the heads at the same time. At the time of servowriting, this alignment is verified. However, if the file is clamped incorrectly in the servowriter or in the final product configuration, the actuator and disk spindle will tilt, resulting in radial misalignment and timing shifts from head to head. Sector servo files are designed to allow for some radial and timing misalignment, but typically these numbers are on the order of $5 \mu\text{m}$ and 500 ns , respectively.

4) *Environment*: The key to successful servowriting is cleanliness and the precise placement of servo information. For this reason, the environment of the servowriter is extremely important. A servowriter should be in a clean room so that exposed files will not become contaminated with dirt particles. Even in a clean room, though, dirt will accumulate on any surface (although minimal). A simple cover should be placed over each open file in the clean room while it is waiting for an assembly operation or servowriting. During the servowriting operation the cover can be removed temporarily. Besides keeping dirt off of the disks and out of the head-disk assembly (HDA) enclosure before it is sealed, magnetic particles should be purged from the clean room. Temperature is critical in precision placement of the tracks. The file dimensions will shift as the temperature changes, so it is best to keep the temperature between 68 and 72°F to minimize these mechanical changes and to facilitate operator comfort. Humidity should be between 40 and 60 percent. Above 60 percent, the humidity causes the servowriter to rust. Below 40 percent, static electricity begins to cause problems.

V. CONCLUSIONS

Servowriters may seem complex on the surface because many interleaved subsystems are involved. However, each subsystem is easy to understand. Because of the building block approach, the servowriter is very logically constructed. It is a precision machine that should be treated with care. Time and money spent on the development of a servowriter usually are recovered in a less expensive

product. However, a servowriter cannot be designed and developed independently of the file. The servowriter's mechanical clamping and access systems are directly tied to the mechanical design of the file. Likewise, a file may behave very poorly after servowriting if its design has not been thoroughly verified on the servowriter. However, no matter how much planning and testing are performed, inevitable adjustments to the file and servowriter do occur during development. They usually come in the areas of vibration isolation, file alignment, and actuator positioning. For this reason, mechanical design considerations in the file and servowriter should be given high priority. Enhancing either the software or the electronics alone will not compensate for mechanical design deficiencies. The environment causes problems that are not directly traceable. Magnetic particle contamination may appear once the product has left the servowriter. Errors in servowriting induced by temperature shifts or head crashes due to accumulation of debris never directly point to the cause, thus requiring anxious search for solutions, usually later during high-volume production. With care, it can be an extremely powerful analysis tool, and without it, a never-ending source of problems.

APPENDIX: SERVO PATTERNS

The phase encoded servo pattern (or phase modulated servo) works on the principle of varying the relative phase of two bursts of squares waves (both at the same frequency) for each successive track. As a head crosses tracks, the phase shift between burst 1 and burst 2 for each track changes. This phase shift becomes an indicator of track position. In Fig. 6(a) the vertical dashed line represents the boundary between burst 1 and burst 2. When the head is positioned over track $N + 1$, there will be no phase shift between burst 1 and burst 2. When the head moves to track $N + 2$, there is a 90° phase shift between burst 1 and burst 2. The phase shift varies quite linearly with position for all head locations between tracks $N + 1$ and $N + 2$. Note that the pattern repeats every four tracks so that the same phase shift results when the head is over track $N, N + 4, N + 8, \dots$. The benefits of a phase encoded pattern are that the servo signal frequency does not vary, the pattern is linear over four tracks, and single bit disk defects usually do not have a significant effect on the servo position signal, since multiple cycles are averaged.

The amplitude servo scheme is more commonly employed today. It has good linearity over track centers, but becomes nonlinear as the head moves off track center by more than 35 percent of the track width. This scheme is usually easier to demodulate and to servowrite. Amplitude

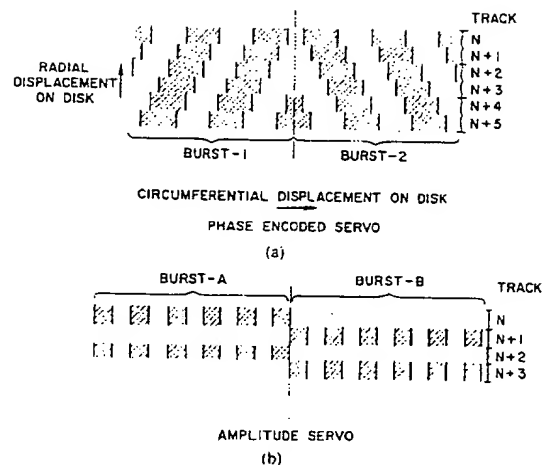


Fig. 6. (a) Phase encoded servo pattern. (b) Amplitude encoded servo pattern.

tude servo relies on the principle that signal amplitude read by a head varies linearly as the head moves off track center. Referring to Fig. 6(b), the read-back signal amplitude from the head will be at its maximum during burst A when the head is positioned over track $N, N + 2, N + 4, \dots$. As the head moves off track, the amplitude of the burst A signal decreases in proportion to the amount of off-track position, and burst B amplitude begins to increase from 0. At tracks $N + 1, N + 3, N + 5, \dots$, the signal from burst B is at its maximum and burst A is zero. Like the phase encoded servo, this scheme will tolerate small disk defects since many cycles are used in the burst. However, amplitude servo is not as linear as a phase encoded servo and there are two widely separated frequency components in the pattern—dc and the burst frequency.

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The Hinf model matching approach to motor tracking control system

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Index Terms:

DC motor drives H^∞ control H^∞ interpolation theory H^∞ model matching algorithm H^∞ tuning controller MISO system PC-based microcomputer implementation Riccati

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The Hinf Model Matching Approach to Motor Tracking Control System

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ABSTRACT

The H^∞ model matching algorithm is proposed for the robust motor tracking control of linear time-varying multi-input, single-output uncertain system. This control method is verified by the system of the permanent magnet DC motor to improve the drive performance and reduce sensitivity of parameter variations, nonlinear effects, and other disturbances such as load changes and backlash. The matching model is used to identify the H^∞ tuning controller with bounded noise energy and minimized tracking error. An algorithm based on discrete Riccati equation and H^∞ interpolation theory is used to solve the uncertain internal modeling and noise measurement issues in the tracking control system. The adaptive tuning controller effectively controls the presence of external disturbances, and identifies the rapid jumping and slow changing trajectory of the system. The effectiveness of the H^∞ model matching controller for the motor tracking control system is demonstrated by using the developed H^∞ adaptive tuning software controller. The H^∞ model matching control method is examined through computer simulation, and then tested in a real time microprocessor-controlled drive system. Experimental results based on PC based microcomputer implementation are presented to illustrate improved response and robust performance.

Keywords: H^∞ model matching, tracking control-system, bounded noise, adaptive tuning.

1. INTRODUCTION

Following the output of a system track from a given reference trajectory is a common industrial problem.

Sometimes the mechanical load varies considerably, but the trajectory of the motor drive still has to follow the required tracking operation. These situations can be found in typical examples, such as: robot joints, and machine tools[1][2][8][9][10]. All these motion systems require the motor bore the load changing and still keep the prescribed trajectory.

The motion disturbance in the above description accords to the uncertainty noise under modeling system which has to be minimized. It is the aim to make the motor drive system to meet the robust stability of control performance. And in order to obtain the satisfactory tracking performance[7], the adaptive tuning controller is usually considered in the linear on-line control strategy for a successful application[1]. However, it needs to design a reasonably accuracy model, or the resulting control algorithm is hard to maintain the tracking system in the robust stability. Envisioned the performance exploited by robust stable modeling operation in the sense, it is believed that the controller should be designed to have abilities to bound the noise disturbance and minimize the tracking error.

In this paper, a motor tracking control design using H^∞ model matching is proposed. It is designed to exclude the noise spectral densities whereas the conventional approaches can not. The merits of the approach lie in the simplicity of the scheme and its practicality for real-time implementation.

The identification has to be carried out within internal model matching condition. In order to make sure that a control transient can be concluded already with the desired response, the identification and the adaptation of the controllers have to be performed as fast as possible. The limited time needed for updating the controllers is determined by the convergence time of the identification algorithm and the time needed to calculate the new controller setting.

Fig. 1-1 is the block scheme diagram of main sections. Brushless dc motor represents in the dotted box. Inverted commutation circuit shows within the dashed line. The others of the diagram are included with the controller. Two input signals of the system from the current sensing which is proportional to the motor voltage and the increment encoder determine the rotor position

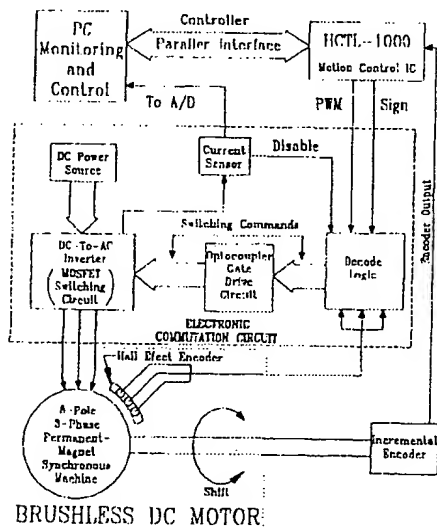


Fig.1-1 The hardware scheme diagram of motor main system

II. DYNAMIC MODEL OF PERMANENT MAGNET MOTOR DRIVE SYSTEM

The permanent magnet motor drive can be regarded as direct current by having d-q axes transformation that rotates synchronously with the alternating current. Because the relative velocity between the rotor shaft and

$$\begin{bmatrix} E_d \\ E_q \end{bmatrix} = \begin{bmatrix} R_a + p L_a & -\omega_m L_a \\ \omega_m L_a & R_a + p L_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \omega_m \phi \\ 0 \end{bmatrix} \quad (2-1)$$
$$di_d/dt = -R_d/L_{di_d} - \phi \omega_m/L_n + E_d/L_n \quad (2-2)$$
$$T_l = J_l (\ddot{\theta}_m + B_m \omega_m + k \phi_d) \quad (2-3)$$

R_n : the armature resistance
 L_n : the armature inductance
 J_I : the motor moment of inertia
 ϕ : the magnetic flux of the permanent magnet
 (constant).
 ω_m : the angular velocity of the motor
 B_m : the coefficient of viscous friction
 $\rho = d/dt$
 T_l : the load torque
 E_d, E_q, i_d and i_q are d, q shaft voltages and currents.

The dynamic model of permanent magnet DC motor is well established in the continuous time domain by the state space model[13]:

$$\dot{x}(t) = Ax(t) + BU(t) \quad (2-4)$$

where

$$\mathbf{x}(t) = [\theta_m(t) \quad \omega_m(t) \quad i_d(t) \quad i_q(t)]$$

$$U(t) = [T_1(t) \quad E_d(t) \quad E_q(t)]$$

$$\underline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -B_m/J_I & k_T/J_I & 0 \\ 0 & k_E/L_a & -R_a/L_a & 0 \\ 0 & 0 & 0 & -R_a/L_a \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 & 0 & 0 \\ -1/J_I & 0 & 0 \\ 0 & 1/L_a & 0 \\ 0 & 0 & -1/L_a \end{bmatrix}$$

where

$x(t)$ = state vector of the motor components.

$U(t)$ = the disturbance (load torque) and the control signal (voltage)

A and B are system matrices corresponding to the input/output vectors, respectively.

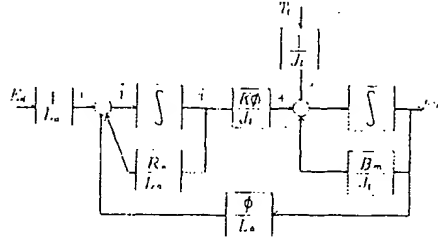


Fig. 2-1 Block diagram of a brushless DC motor

To protect the motor from overload in the motor tracking system. The motor system has specified constraints on the motor current. These limitations are as follows[2][7][11]:

(1) Peak current limitation: The maximum occurring motor is limited to avoid demagnetization of the permanent magnets.

(2) Continuous current limitation: To prevent the motor from overheating, the maximum constant current is limited to approximately half the peak current for the motor used. The continuous current limitation becomes active only after a certain period of time and can be disregarded in position control. However, if torque control is used, longer periods of maximum applied power can occur; in this case, the continuous current limited is important for the prevention of the power capacity of the driver components.

(3) Commutation limitation: The inrush current through the power drive components increases with the motor speed and the heavy load variation. To prevent the motor from being damaged, the product of armature current and voltage is limited.

In this paper, however, a systematic method is proposed to determine the plant orders and time delay of the motor tracking system[1][17]. To derive the computation equations, the time domain model of PM machine with a controller is transformed into the discrete model in the z domain form, and generates the system upper bound for the worse case, as well as robust tracking output. The system is specified the requirement as follows:

speed bandwidth: 0 - 50Hz, under load condition and 12.5Hz for loading variation. The robustness is as large as possible within the setting frequency response. The accuracy of the position is within 1um, and no exciting phenomenon. Insensitivity of load disturbance and noisy

interference is designed in the adaptive control scheme and modeling matching.

III. THE H^∞ SYNTHESIS CONTROL CONCEPTS

First, the H^∞ synthesis control system is structured as a standard problem, depicted in Figure 3-1. where the objective is to design a real-rational proper controller K such that the closed-loop system formed by real rational proper G and K is internally stable and the H^∞ norm of the transfer function between external input vector w and controlled output vector z is minimized over all stabilizing controllers[3].

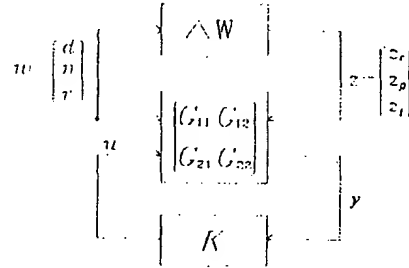


Fig. 3-1 H^∞ synthesis standard problem

w : exogenous input vector included disturbance d , noise n , and reference r .

z : controlled output vector included tracking error z_e , energy z_g , and output tracking z_f .

y : observed measured output vector.

u : controlled input state.

G : The augmented control model.

K : The feedback controller.

This augmented control system G can be written as:

$$G: G = C(qI - A)^{-1}B + D \quad (3-1)$$

The synthesis system can be represented as:

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} z \\ y \end{bmatrix} \quad (3-2)$$

Setting the system gain K for feedback as:

$$u = Ky \quad (3-3)$$

The convergence and stability of the system can be obtained from inputs w to the outputs z of the close loop function. The form is expressed as:

$$z = T_{zw}(G, K)w \quad (3-4)$$

$$T_{zw} = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \quad (3-5)$$

Eqn(3-5) represents the standard form of H^∞ synthesis control system. The augmented synthesis system has to find the controller K so that

$$\inf_K \|G_{11} + G_{12}K(J - G_{22}K)^{-1}G_{21}\|_\infty \quad (3-6)$$

The controller K [4] has to satisfy the following two conditions:

(i) The closed loop system is internally stable.

$$(ii) \min \|T_{zw}(G, K)\|_\infty < \gamma \quad 0 < \gamma < 1$$

The augmented control system G means the combination of the real control system P and filter functions ΛW for the tracking error and the internal system entropy. The filter function we adopt in this article shall consider the response of the output tracking as well as the minimized sensitivity for the system stability thereby boosting the system's performance and robustness.

The synthesis H^∞ system is examined in reconstruction state space forms[5][6]. The mean square reconstruction error value is to measure the observer's reconstruction status. To design a time varying tuning procedure as follows:

$$Q(n+1) = \hat{A}(n)Q(n)\hat{A}^T(n) + W_1(n) \quad (\text{pre-covariance}) \quad (3-7)$$

$$L(n+1) = Q(n+1)\hat{C}^T(n+1)[\hat{C}(n+1)Q(n)\hat{C}^T(n+1) + W_2]^{-1} \quad (\text{state estimation gain}) \quad (3-8)$$

$$\hat{Q}(n+1) = [I - L(n+1)\hat{C}(n+1)]Q(n) \quad (\text{observed covariance}) \quad (3-9)$$

From the above adaptive procedure, the synthesis H^∞ system shall be examined from the predictor-corrector state.

$$\begin{aligned} \hat{x}(n+1) &= \hat{A}(n)\hat{x}(n) + \hat{B}(n)u(n) + L(n+1)\{ \\ y(n+1) - \hat{C}(n)[\hat{A}(n)\hat{x}(n) + \hat{B}(n)u(n)] \} \end{aligned} \quad (3-10)$$

IV. THE ADAPTIVE MODELING IDENTIFICATION

The dynamic model of PM DC motor is well set up by the augmented system that is translated into the difference equation necessary for use in the discrete system[15].

The discrete form is given by eqn (4-1) in the state space model:

$$\bar{x}_t = \bar{A}(n-1)\bar{x}(n-1) + \bar{B}(n-1)\bar{u}(n-1) \quad (4-1)$$

The matrices $\bar{A}(n-1)$, and $\bar{B}(n-1)$ in discrete-time model are based on the motor's parameters in eqn (2-4).

The approach to make the identification of a motor trajectory employs the QR based method to solve the recursive least square(RLS) problem. The QR based method is numerically stable. It shows that the

computation method of matrix problems is for orthogonal similarity transformation.

To do so in the adaptive tuning procedure[17], the objective is to finally obtain a predictor model for the desired tracking y_t :

$$y_t(n) = X^T(n)\theta(n) + \zeta_t \quad (4-1)$$

where $X^T(n)$ is an observed vector using past values of y_t and X . $\theta(n)$ is a parameter vector using the coefficients from the above of $A(n-1)$ and $B(n-1)$, and ζ_t is an error variable representing the load disturbance and modeling error. From the QR based least squares computation, it is to see that generation of pseudo-linear variable y'_t , defined as

$$y'_t = \gamma^{-1}AT^TX(n) \quad (4-2)$$

The setting up recursive least square (RLS) model, keep the identification order low. A state space predictor for the plant model[14] can now be set up by a series of estimation steps.

$$\text{error: } e_t = y'_t - X(n)\hat{\theta}(n) \quad (4-3)$$

controller gain:

$$K_t(n) = -[\hat{B}^TP_{t-1}(n)\hat{B} + \lambda_{t-1}(n)]^{-1}\hat{B}^TP_{t-1}(n)\hat{A} \quad (4-4)$$

$$\text{estimate: } \hat{\theta}_t(n) = \hat{\theta}_{t-1}(n) + K_{t-1}(n)e_t \quad (4-5)$$

forgetting factor:

$$\lambda_t(n) = I - (I + \hat{B}^TP_{t-1}(n)\hat{B})e_t^2/\Gamma \quad (4-6)$$

covariance:

$$\begin{aligned} P_t(n) &= [\hat{A} + \hat{B}K_t(n)]^TP_{t-1}(n)[\hat{A} + \hat{B}K_t(n) \\ &+ K_t^T(n)\lambda_t(n)K_t(n)] \end{aligned} \quad (4-7)$$

$P_t(n)$ has to be positive semidefinite, and if $P_t(n)$ has rank defect r , then it needs to use an orthogonal transformation T to adjust it.

V. The Model Matching for the Motor System

The estimator of the system is reconstructed after every periodic computation. It consists of a set of time varying equations from eEqn (3-7) to eqn (3-10). These works can form an estimated system model. The controller of the augmented system is designed in accordance with eqn (4-2) to eqn (4-7). It executes in the state of the adaptive tuning to minimize the motor tracking error.

The H^∞ approach design is needed to specify the disturbance from the input for the worse case. Therefore, the term gamma γ is required to setup for the robust range.

$$\sigma(P_t(n)Q(n)) < \gamma^2 \quad (5-1)$$

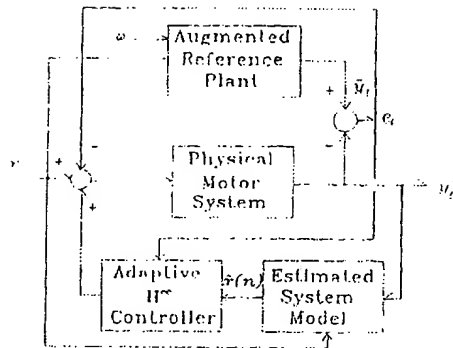


Fig. 5-1: The H^∞ synthesis model control system

It means that the spectral radius of $P_i(n)Q(n)$ must be lower than the upper bound of disturbance γ .

Fig. 5-1 shows the H^∞ synthesis control scheme model for the motor control system. From the figure 5-1 the estimated system model is derived from the real motor system and the desired augmented reference plant. The adaptive H^∞ controller is designed after the system model [16] is estimated.

VI. TEST RESULTS

Several test cases in the experiment evaluate the proposed H^∞ approach model matching technique.

The position track selected in this test is composed of three segments of sigmoidal functions. The first rotates the forward position by 200 radian, the second is holding the position at 200 radian for 2 second, and the third turns the reverse direction back to the original position by the armature current braking from the feedback sensor signal. In these test cases, the armature surge current is limited within the specified range to verify the motor system running within the designed tracking and safety working condition.

Two cases are shown on the following mainly to represent the trajectories of the motor rotor position and required speed tracking paths. Each of the motor system tracking is measured from the starting time and sampled in every 20 msec.

Case I shows figures from Fig. 6-1 to Fig. 6-10 to examine the motor system performance and robust. Case I represents 10% perturbation with the required trajectory entering the motor system and the load torque changing periodically every 12.5Hz. Figure 6-1 to 6-7 shows the tracks and the measurements of the rotor position, speed, voltage, and electric torque, position tolerance, direct current, and quadrature current, respectively. The

dotted line on Fig. 6-1 and Fig. 6-2 represents the desired trajectory, while the dash line represents the output trajectory. The position trajectory follows the desired sigmoidal tracking on Fig. 6-1. The speed trajectory gets some disturbance but still maintain the set speed

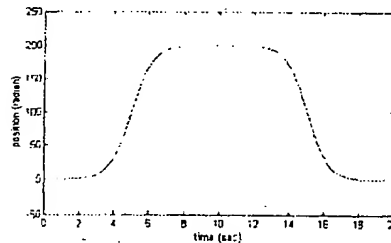


Fig. 6-1 Case I: Position trajectory

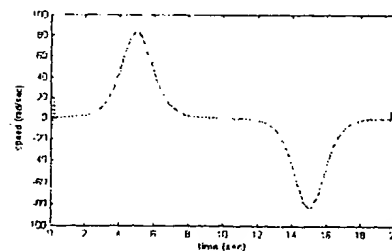


Fig. 6-2 Case I: Speed trajectory

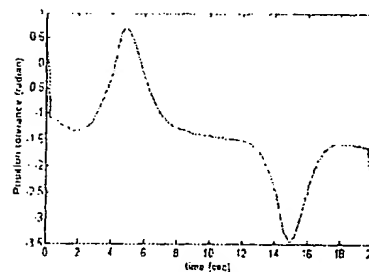


Fig. 6-3 Case I: Position tolerance

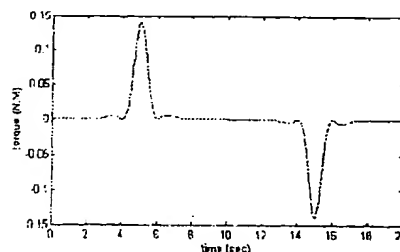


Fig. 6-4 Case I: Electric torque

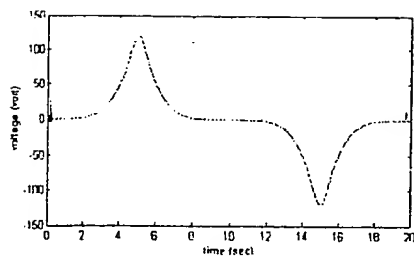


Figure 6-5 Case I: Armature voltage

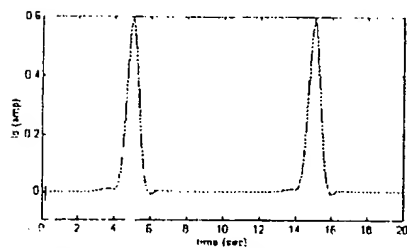


Fig. 6-6 Case I: Direct current

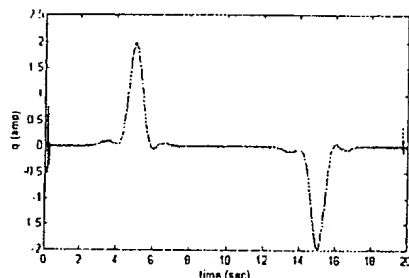


Fig. 6-7 Case I: Quadrature current

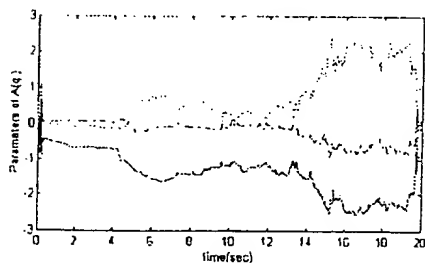


Fig. 6-8 Case I: Parameters of $\bar{A}(q)$

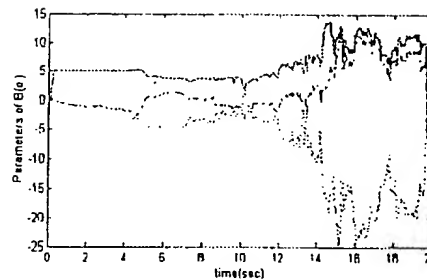


Fig. 6-9 Case I: Parameters of $\bar{B}(q)$

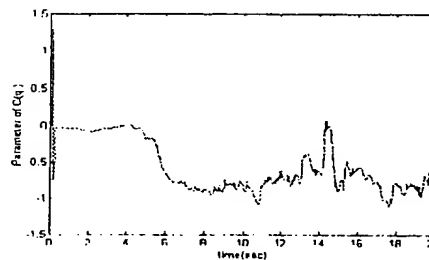


Fig. 6-10 Case I: Parameter of $\bar{C}(q)$

running on Fig. 6-2. Fig. 6-4 shows the electric torque increased during the motor accelerating the speed. However, the motor system still maintains in the safe running condition by examining the voltage and current variation from Fig. 6-5 to Fig. 6-7.

The time varying motor system under the noise perturbation and load change highly depends on the adaptive tuning for the stable and robust synthesis system performance. Fig. 6-8 to Fig. 6-10 show the synthesis parameters of the H^∞ system. The parameters of matrices of $\bar{A}(q)$, $\bar{B}(q)$ and $\bar{C}(q)$ show the adaptive identification for the reduction order H^∞ approach matching. Specifically, when the motor proceeds the tracking in the braking segment, the system parameters vary quite largely.

To satisfy the robust performance of the H^∞ synthesis approach, it is designed the upper bound for the worse case at the gamma value (γ) 0.67 in Case I.

Case II represents the situation of the 50% uniform distributed perturbation with the desired trajectory entering the motor system and the load torque changing periodically every 25Hz. Fig. 6-11 to Fig. 6-15 shows the performance of the motor system in case II. It examines the motor system with the robust control in the H^∞ synthesis approach. In Case II, the gamma term γ is designed 0.45 for the robust range. It can meet the case to limit the upper bound of disturbance.

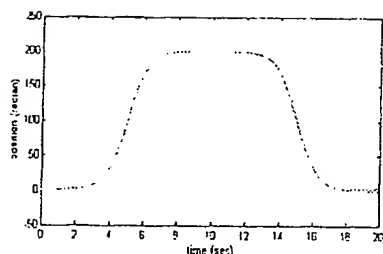


Figure 6-11 Case II: Position trajectory

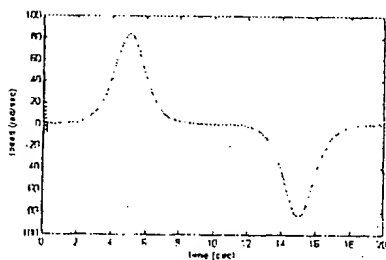


Fig. 6-12 Case II: Speed trajectory

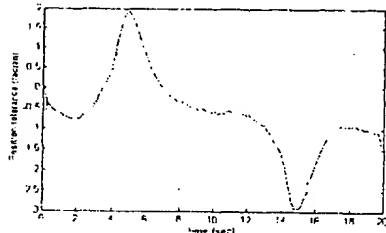


Figure 6-13 Case II: Position tolerance

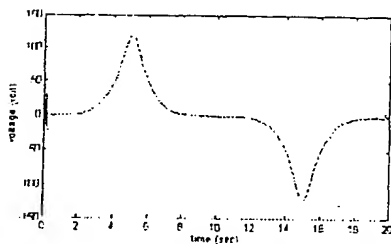


Fig. 6-14 Case II: Armature voltage

The dotted line on Fig. 6-11 and Fig. 6-12 represents the desired trajectory, while the dash line represents the output trajectory. Fig. 6-11 shows the position trajectory following the desired tracking under the heavy

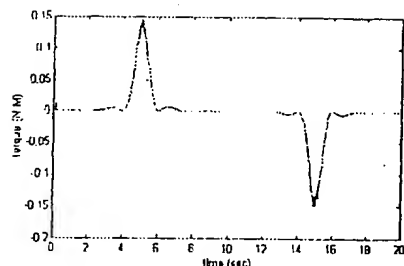


Fig. 6-15 Case II: Electric torque

noise and load varying. It can be seen from Fig. 6-13 that the position trajectory has one step predicted error, especially, during the motor increasing speed. However, the dc error of the position trajectory is derived from the uncertainty disturbance. It can be compensated to adjust the deadbeat of the filter.

In this paper, it mainly emphasizes the performance of the position trajectory. Therefore, the speed trajectory on Fig. 6-12 is infected by some disturbance similarly as Case I, during the motor starting, speedup, and braking stop. Fig. 14 and Fig. 15 show the motor system still stable when the strong perturbation adds to the system. In Case II it still can work within the protected safety range.

Fig. 6-13 shows that the larger position tolerance appears as the disturbance become larger, comparing with Case I. It shows that the dc tolerance is reduced by adding the constant backlash. However, it is designed from the synthesis system to bound the disturbance in the spectral radius.

VII. CONCLUSIONS

In this paper, the design and implementation of an H^∞ matching controller with an adaptive tuning has been investigated. A synthesis systematic and analysis approach for developing the algorithm is presented. The H^∞ filter is tuned by various parameters $Q(n)$, $L(n)$, $K(n)$, $\lambda(n)$, $P_f(n)$, and $X(n)$ which are dominant in the state identified model and the designed H^∞ controller. These parameters have to meet the desired transient and steady state performance. The discrete adaptive H^∞ matching controller has been designed to be well suited the motor speed and position trajectory for Brushless DC motor. The load torque variation and mechanical damper can be immunize by the designed H^∞ controller under the upper bound area. The next step in this research will test the H^∞ matching

controller in different kinds of disturbance for the speed and position trajectory.

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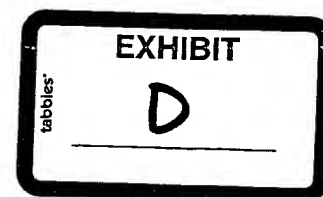
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Nonlinear state observation using H_∞ -filtering Riccati design

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Abstract:

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Index Terms:

[Kalman filters](#) [Lyapunov methods](#) [Riccati equations](#) [continuous time systems](#) [differential equations](#) [filtering theory](#) [induction motors](#) [nonlinear systems](#) [observers](#) [robust control](#) [\$H_\infty\$ -filtering](#) [Lyapunov function](#) [Riccati differential equation](#) [angular velocity](#) [continuous-time systems](#) [extended Kalman filter](#) [induction motor](#) [nonlinear systems](#) [observer](#) [robust control](#) [rotor flux](#) [state observation](#)

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Nonlinear State Observation Using H_∞ -Filtering Riccati Design

Konrad Reif, Frank Sonnemann, and Rolf Unbehauen

Abstract— In this paper, the authors propose an observer for continuous-time nonlinear systems. The observer gain is computed by a Riccati differential equation similar to the extended Kalman filter. They prove that under certain conditions the proposed observer is an exponential observer by choosing an appropriate Lyapunov function. Furthermore, the authors explore some important relations of the proposed observer to robust control theory and H_∞ -filtering. To examine the practical usefulness of the proposed observer they applied it to an induction motor for the estimation of the rotor flux and the angular velocity.

Index Terms— Extended Kalman filter, H_∞ -filtering, nonlinear observer.

I. INTRODUCTION

The basic model for the general nonlinear observation problem consists of a nonlinear plant and a nonlinear output map, at which neither system noise nor observation noise is considered. The problem is to estimate the states of the plant by observing the output when the initial state of the plant is not known. In literature the design of nonlinear observers is a widespread area of current research (see [13] and [22] for a survey). Although many important results have been achieved, there are still possibilities for an improvement of the existing synthesis methods.

A frequently applied method is the extended Kalman filter [5], used as a deterministic observer for nonlinear systems. The advantage is its easy handling with a simultaneous good performance; moreover, a full proof of its exponential stability has been given recently [16], [18]. However, a well-known difficulty arising through its application is the problem of divergence. The reason is that in computation of the observer gain only the linearized functions are considered, but not the whole nonlinearities. To obtain convergence, it is therefore, in general, necessary either to have a good initial guess so that the initial estimation error is sufficiently small or to permit functions only weakly nonlinear [16], [18].

Recently H_∞ -control methods have attracted attention in robust control theory [4], [11]. It has been mentioned that the high robustness of the LQ-optimal control systems can be further improved by using H_∞ -control [3]. In this paper we use a similar approach to improve the extended Kalman filter. Motivated by the results of robust control theory [9], [20] and H_∞ -filtering [2], [6], [10], [14]. The remaining nonlinear terms are treated as uncertainties and are implemented in the proposed observer design by a worst case valuation in such a way that these uncertainties can be tolerated. Consequently, better convergence properties can be expected; moreover, under certain conditions convergence can be achieved even independent of the initial estimation error. An alternative method for improving the

convergence properties of the extended Kalman filter has been proposed in [17].

The paper is organized as follows: in Section II we state the necessary preliminaries and introduce the proposed nonlinear observer. By choosing an appropriate Lyapunov function we show that the proposed observer is an exponential observer in the sense of [8]. Important relations to H_∞ -filtering [2], [6], [10], [14] are explored in Section III. In Section IV we apply the proposed observer to estimate the rotor flux and the angular velocity of an induction motor (cf. [12], [15], [19]). Some conclusions are drawn in Section V.

II. PROPOSED OBSERVER

Consider a nonlinear system represented by the equations

$$\dot{z}(t) = f(z(t), x(t)) \quad (1)$$

$$y(t) = C z(t) \quad (2)$$

where $z(t) \in R^q$ is the state, $x(t) \in R^p$ the input, $y(t) \in R^m$ the output, and C a $m \times q$ matrix. The function $f(\cdot, \cdot)$ and the input $x(\cdot)$ are assumed to be C^1 -functions. For the sake of simplicity we are restricted to linear time-invariant readout maps. The important case where some of the state variables can be measured directly is thereby treated. Furthermore, the linear time-invariant readout map is well suited to pointing out the main idea. Some remarks on the more general case with nonlinear readout maps are given in Section III. For the considerations following below we need a slightly more general definition of the exponential observer in [8].

Definition: For a dynamic system given by (1) and (2) an exponential observer is a dynamic system

$$\dot{\hat{z}}(t) = f(\hat{z}(t), x(t)) + g(y(t), \hat{z}(\cdot), x(\cdot), t) \quad (3)$$

with an output injection $g(\cdot, \cdot, \cdot, \cdot)$ such that

$$\|z(t) - \hat{z}(t)\| \leq \eta \|z(0) - \hat{z}(0)\| e^{-\theta t} \quad \forall t \geq 0, \quad \forall z(0) - \hat{z}(0) \in B_\rho \quad (4)$$

holds for some $\rho, \eta, \theta \in R^+$ with $B_\rho = \{v \in R^q : \|v\| < \rho\}$.

Remark: The output injection $g(\cdot, \cdot, \cdot, \cdot)$ depends in general on the whole functions $\hat{z}(\cdot)$ and $x(\cdot)$ and not only on the recent values $\hat{z}(t)$ and $x(t)$. The reasons are given below.

To calculate the output injection $g(\cdot, \cdot, \cdot, \cdot)$ we introduce a Riccati differential equation which is similar to the Riccati differential equation for the extended Kalman filter [5]. Using the abbreviation

$$A(t) = \frac{\partial f}{\partial z}(\hat{z}(t), x(t)) \quad (5)$$

it reads

$$\begin{aligned} \dot{P}(t) = & A(t)P(t) + P(t)A^T(t) \\ & - P(t)(C^T R^{-1} C - \lambda^2 I)P(t) + \mu^2 I \end{aligned} \quad (6)$$

with some $\lambda, \mu \in R^+$ and a positive definite symmetric matrix R . We define the observer gain by

$$K(t) = P(t)C^T R^{-1} \quad (7)$$

and the output injection by

$$g(y(t), \hat{z}(\cdot), x(\cdot), t) = K(t)(y(t) - C\hat{z}(t)). \quad (8)$$

It is necessary to allow a general dependence on $\hat{z}(\cdot)$ and $x(\cdot)$ and not only on the recent values $\hat{z}(t)$ and $x(t)$ because the solution

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of the Riccati differential equation (6) depends on past values of $A(t) = (\partial f / \partial z)(\hat{z}(t), x(t))$.

An expansion of $f(\cdot, \cdot)$ at $\hat{z}(t), x(t)$ leads to

$$\begin{aligned} f(z(t), x(t)) - f(\hat{z}(t), x(t)) \\ = A(t)(z(t) - \hat{z}(t)) + \varphi(z(t), \hat{z}(t), x(t)) \end{aligned} \quad (9)$$

where $\varphi(z(t), \hat{z}(t), x(t))$ are the terms of second- and higher order in $z(t) - \hat{z}(t)$. Defining the estimation error by

$$\zeta(t) = z(t) - \hat{z}(t) \quad (10)$$

subtracting (3) from (1) and considering (8) and (9), we obtain

$$\dot{\zeta}(t) = (A(t) - K(t)C)\zeta(t) + \varphi(z(t), \hat{z}(t), x(t)). \quad (11)$$

For an exponential observer this differential equation has an exponentially stable equilibrium point at zero; moreover, if (4) holds for all $\zeta(0) \in R^q$ the equilibrium point is globally exponentially stable (cf. [21, Sec. 5.1, p. 143]).

To state the proof of the Main theorem given below we need the following preparation.

Lemma: Consider real vectors $z, \hat{z} \in R^n$ and $x \in R^p$, a symmetric $q \times q$ matrix Π , and a function $\varphi(\cdot, \cdot, \cdot)$ with

$$\|\varphi(z, \hat{z}, x)\| \leq \kappa \|z - \hat{z}\| \quad (12)$$

for some $\kappa \in R^+$. Choose $\lambda, \mu \in R^+$ such that $\kappa < \lambda\mu$ holds. Then there is a $\delta > 0$ such that

$$2(z - \hat{z})^T \Pi \varphi(z, \hat{z}, x) \leq (z - \hat{z})^T [\lambda^2 I + (\mu^2 - \delta) \Pi \Pi] (z - \hat{z}) \quad (13)$$

holds.

Proof: From $\kappa < \lambda\mu$ it follows that there is a $0 < \delta' < \lambda\mu$ such that

$$\kappa < \lambda\mu - \delta'$$

holds. Inserting into (12) yields

$$\|\varphi(z, \hat{z}, x)\| \leq (\lambda\mu - \delta') \|z - \hat{z}\|$$

and therefore we have

$$2(z - \hat{z})^T \Pi \varphi(z, \hat{z}, x) \leq 2(\lambda\mu - \delta') \|\Pi(z - \hat{z})\| \|z - \hat{z}\|. \quad (14)$$

For considerations following below we evaluate the square

$$\begin{aligned} & \left(\left(\mu - \frac{\delta'}{\lambda} \right) \|\Pi(z - \hat{z})\| - \lambda \|z - \hat{z}\| \right)^2 \\ &= \left(\mu^2 - \frac{2\mu\delta'}{\lambda} + \frac{\delta'^2}{\lambda^2} \right) (z - \hat{z})^T \Pi \Pi (z - \hat{z}) \\ & \quad - 2\lambda \left(\mu - \frac{\delta'}{\lambda} \right) \|\Pi(z - \hat{z})\| \|z - \hat{z}\| + \lambda^2 (z - \hat{z})^T (z - \hat{z}) \geq 0 \end{aligned}$$

and use the fact that it takes only nonnegative values. Rearranging the terms yields

$$\begin{aligned} & 2(\lambda\mu - \delta') \|\Pi(z - \hat{z})\| \|z - \hat{z}\| \\ & \leq (z - \hat{z})^T \left[\left(\mu^2 - \left(\frac{2\mu\delta'}{\lambda} - \frac{\delta'^2}{\lambda^2} \right) \right) \Pi \Pi + \lambda^2 I \right] (z - \hat{z}). \end{aligned} \quad (15)$$

Choosing

$$\delta = \frac{2\mu\delta'}{\lambda} - \frac{\delta'^2}{\lambda^2}$$

from $\delta' < \lambda\mu$ it follows that $\mu > \delta'/\lambda$ and therefore

$$\delta > 2 \frac{\delta'}{\lambda} \frac{\delta'}{\lambda} - \frac{\delta'^2}{\lambda^2} > 0.$$

Combining (14) and (15) we obtain the desired result (13). \square

With these preliminaries we are able to state Theorem 1.

Theorem 1: Consider a nonlinear system with linear time-invariant readout map given by (1), (2) and an observer given by (3) and (6)–(8) with some $\lambda, \mu > 0$ and a positive definite matrix R , such that the following conditions hold.

- 1) For any solution $\hat{z}(\cdot)$ of the observer differential equation (3) the solution $P(\cdot)$ of the Riccati differential equation (6) is bounded via

$$\underline{p}I \leq P(t) \leq \bar{p}I \quad (16)$$

for some $\underline{p}, \bar{p} > 0$.

- 2) The nonlinear terms $\varphi(z(t), \hat{z}(t), x(t))$ in the differential equation (11) for the estimation error are bounded via

$$\|\varphi(z(t), \hat{z}(t), x(t))\| \leq \kappa \|z(t) - \hat{z}(t)\| \quad (17)$$

with $\kappa < \lambda\mu$.

Then the considered observer is an exponential observer. Moreover, the differential equation (11) for the estimation error is globally exponentially stable at zero.

Remarks:

- 1) To ensure the global exponential stability of the differential equation (11) we require that (17) is satisfied in the whole state space. If this inequality holds only in a restricted area, only local exponential stability can be guaranteed. This means that in this case the initial estimation error may not be arbitrary large. Nevertheless, numerical simulations have shown that even if the stability is not globally satisfied, the proposed observer can tolerate much higher initial estimation errors than an observer based on the extended Kalman filter (see Section IV).
- 2) Using similar techniques as in the proof following below, it can be shown that under certain conditions the continuous-time extended Kalman filter is an exponential observer [16], [18] too. The difference is that the equilibrium point of the estimation error is globally exponentially stable for the proposed observer but not for the extended Kalman filter. This means that the proposed observer can tolerate arbitrary large initial estimation errors, which is not the case for the extended Kalman filter.
- 3) For the usual extended Kalman filter it can be shown that the solution of the Riccati differential equation is bounded via (16), if the matrices $A(t)$ and C fulfill the uniform observability condition [7] or if the nonlinear system satisfies a certain detectability condition as stated in [1]. Unfortunately, these conditions cannot be applied to get similar results for the H_∞ -filtering-like Riccati differential equation (6). (The reason is that the term $C^T R^{-1} C - \lambda^2 I$ is in general not positive definite because $C^T R^{-1} C$ does not have full rank if we treat nontrivial observation problems.) It is a well-known problem arising in H_∞ -control as well as H_∞ -filtering that the solutions of the corresponding algebraic Riccati equations and Riccati differential equations may lack being positive definite (cf. [3]).
- 4) For the proof below as well as for the lemma stated above, the parameter μ does not play any important role. It has been introduced to have an additional degree of freedom during the implementation process. Especially for ensuring Condition 1, the additional design parameter μ has turned out to be very useful. Also see the following remark.
- 5) The existence of a solution $P(\cdot)$ for the Riccati differential equation (6) which satisfies Condition 1 depends mainly on an

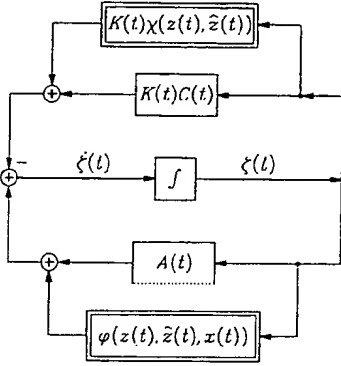
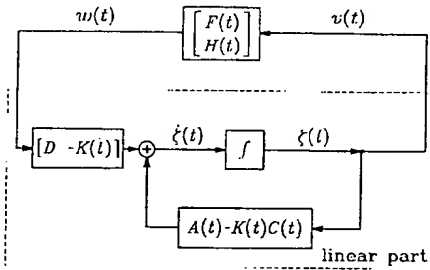


Fig. 1. Block diagram corresponding to the differential equation (27).

Fig. 2. Block diagram corresponding to the differential equation (31). The whole system consists of two interconnected subsystems. The dashed box contains the linear part of the system and the observer; $F(t)$ and $H(t)$ represent the higher order nonlinear terms.

appropriate choice of λ and μ . As will be shown in Section III, they are closely related to the attenuation constant γ in H_∞ -filtering problems. To find suitable values for λ and μ we employed a binary search algorithm, which is widely used to solve H_∞ -control and H_∞ -filtering problems (see, e.g., [3]) and adapted it for our purpose (see Section IV).

Proof: We consider the differential equation (11) for the estimation error and prove its exponential stability by choosing the Lyapunov function

$$V(\zeta(t), t) = \zeta^T(t) \Pi(t) \zeta(t) \quad (18)$$

with $\Pi(t) = P^{-1}(t)$. Because of Assumption 1 we have the following bounds for the Lyapunov function:

$$\frac{1}{p} \|\zeta(t)\|^2 \leq V(\zeta(t), t) \leq \frac{1}{p} \|\zeta(t)\|^2 \quad (19)$$

which imply positive definiteness and decreasence. The time derivative of the Lyapunov function reads

$$\dot{V}(\zeta(t), t) = \dot{\zeta}^T(t) \Pi(t) \zeta(t) + \zeta^T(t) \dot{\Pi}(t) \zeta(t) + \zeta^T(t) \Pi(t) \dot{\zeta}(t).$$

Inserting $\dot{\zeta}(t)$ according to the differential equation (11) yields

$$\begin{aligned} \dot{V}(\zeta(t), t) = & \zeta^T(t) \dot{\Pi}(t) \zeta(t) + [(A(t) - K(t)C(t))\zeta(t) \\ & + \varphi(z(t), \hat{z}(t), x(t))]^T \Pi(t) \zeta(t) \\ & + \zeta^T(t) \Pi(t) [(A(t) - K(t)C(t))\zeta(t) + \varphi(z(t), \hat{z}(t), x(t))]. \end{aligned}$$

Applying the lemma and considering (7) we get with $\kappa < \lambda\mu$

$$\begin{aligned} \dot{V}(\zeta(t), t) \leq & \zeta^T(t) [\dot{\Pi}(t) + \Pi(t)A(t) + A^T(t)\Pi(t) \\ & - 2C^T R^{-1}C + (\mu^2 - \delta)\Pi(t)\Pi(t) + \lambda^2 I] \zeta(t). \end{aligned}$$

Using

$$\dot{\Pi}(t) = -\Pi(t)\dot{P}(t)\Pi(t) \quad (20)$$

and the Riccati differential equation (6) leads to

$$\dot{V}(\zeta(t), t) \leq \zeta^T(t) [-\delta \Pi(t)\Pi(t) - C^T R^{-1}C] \zeta(t). \quad (21)$$

With the bounds (16) for $P(t)$ we obtain the inequality

$$\dot{V}(\zeta(t), t) \leq -\frac{\delta}{\bar{\mu}^2} \|\zeta(t)\|^2 \quad (22)$$

which implies that the time derivative of the Lyapunov function is negative definite. With (19) and (22) we satisfy the requirements to apply [21, Th. 62, Sec. 5.3, p. 173] and conclude that the differential equation (11) has a globally exponentially stable equilibrium point at zero. This includes that the considered observer is an exponential observer. \square

III. RELATIONS TO H_∞ -FILTERING AND ROBUST CONTROL THEORY

Considering the small gain theorem the proposed observer is closely related to H_∞ -filtering theory as will be shown below. Since these relations are more obvious for the case with nonlinear readout maps, we let a nonlinear continuous-time system be given by

$$\dot{z}(t) = f(z(t), x(t)) \quad (23)$$

$$y(t) = h(z(t)) \quad (24)$$

with a C^1 -function $h(\cdot)$. The linear readout map is thereby contained as a special case. For this nonlinear system we construct an observer according to (3) and (6)–(8). Using the fact that the functions $f(\cdot, \cdot)$ and $h(\cdot)$ are continuously differentiable, we expand them via (9) and

$$h(z(t)) - h(\hat{z}(t)) = C(t)(z(t) - \hat{z}(t)) + \chi(z(t), \hat{z}(t)) \quad (25)$$

where $C(t)$ is an $m \times q$ matrix given by

$$C(t) = \frac{\partial h}{\partial z}(\hat{z}(t)). \quad (26)$$

Defining the estimation error by (10) we obtain from (23) and (3) using (8), (9), and (25)

$$\begin{aligned} \dot{\zeta}(t) = & (A(t) - K(t)C(t))\zeta(t) + \varphi(z(t), \hat{z}(t), x(t)) \\ & - K(t)\chi(z(t), \hat{z}(t)). \end{aligned} \quad (27)$$

The nonlinear terms are assumed to satisfy (17) as well as

$$\|\chi(z(t), \hat{z}(t))\| \leq \lambda \|z(t) - \hat{z}(t)\| \quad (28)$$

with some $\lambda > 0$. Now we have a closer look at the differential equation (27) for the estimation error. Fig. 1 shows the corresponding block diagram. Defining

$$F(t) = \frac{1}{\mu \|\zeta(t)\|^2} \varphi(z(t), \hat{z}(t), x(t)) \zeta^T(t) \quad (29)$$

$$H(t) = \frac{1}{\|\zeta(t)\|^2} \chi(z(t), \hat{z}(t)) \zeta^T(t) \quad (30)$$

for $\zeta(t) \neq 0$ we have from (27)

$$\dot{\zeta}(t) = (A(t) - K(t)C(t) + DF(t) - K(t)H(t))\zeta(t) \quad (31)$$

with $D = \mu I$ as shown in Fig. 2. Moreover, we obtain

$$\left\| \begin{bmatrix} F(t) \\ H(t) \end{bmatrix} \right\| < \lambda$$

for every $t \geq 0$ which is an immediate consequence of (17), (28)–(30), and $\kappa < \lambda\mu$. By this construction the nonlinearities are considered as time-varying uncertainties. For the stability of (31), we have the following theorem.

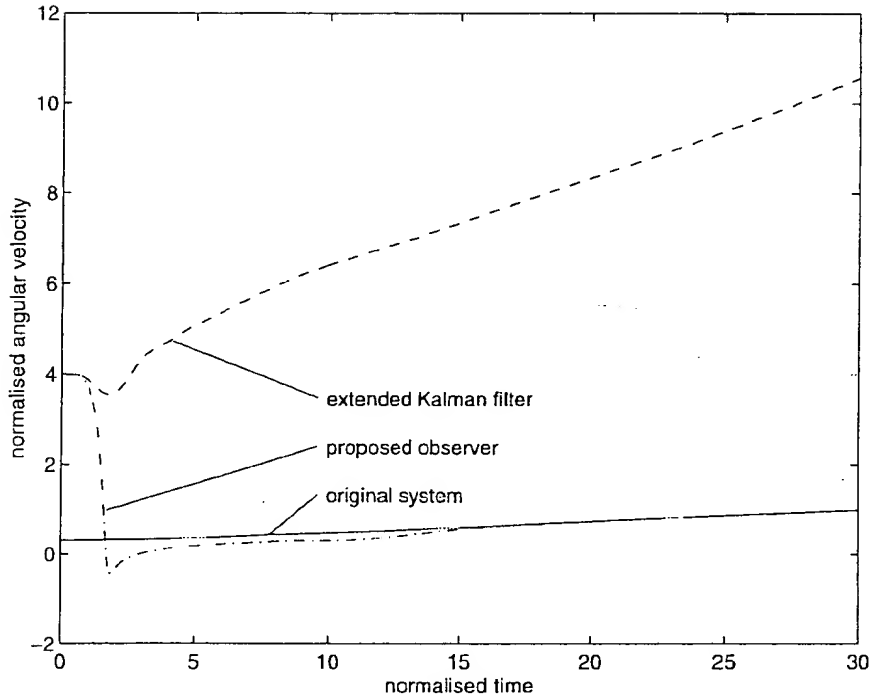


Fig. 3. Normalized angular velocity $z_5(t)$ for the original system, the proposed observer, and the extended Kalman filter with a medium initial estimation error.

Theorem 2: Consider (31) with $D = \mu I$ and an observer gain $K(t)$ given by (6) and (7) with $\lambda, \mu > 0$, and $R = I$. Let the following conditions hold.

- 1) The Riccati differential equation (6) has a solution $P(\cdot)$ which is bounded via

$$\underline{p}I \leq P(t) \leq \bar{p}I$$

for some $\underline{p}, \bar{p} > 0$.

- 2) The matrices $F(t)$ and $H(t)$ are norm-bounded by

$$\left\| \begin{bmatrix} F(t) \\ H(t) \end{bmatrix} \right\| < \lambda$$

for every $t \geq 0$.

Then the system (31) is exponentially stable.

Proof: For the proof of this theorem we employ a variant of the small gain theorem. We recall the operator norms [10] for the two subsystems in Fig. 2 (e.g., $[F^T(t) \ H^T(t)]^T$ and the part inside the dashed box)

$$\|T_{wv}\|_\infty = \sup_{\substack{v \in L_2 \\ \|v\|_2 \neq 0}} \frac{\|w\|_2}{\|v\|_2}, \quad \|T_{vw}\|_\infty = \sup_{\substack{v \in L_2 \\ \|v\|_2 \neq 0}} \frac{\|v\|_2}{\|w\|_2}$$

where $\|\cdot\|_2$ denotes the usual L_2 -norm for functions. According to [20, Corollary 1], the whole system in Fig. 2 is exponentially stable if the subsystem in the dashed box is exponentially stable and the small gain condition

$$\|T_{vw}\|_\infty \|T_{wv}\|_\infty < 1 \quad (32)$$

is satisfied. The operator norm $\|T_{vw}\|_\infty$ can be estimated easily to

$$\|T_{vw}\|_\infty = \sup_{t \geq 0} \left\| \begin{bmatrix} F(t) \\ H(t) \end{bmatrix} \right\| < \lambda. \quad (33)$$

Moreover, it can be seen that the subsystem inside the dashed box in Fig. 2 has the structure of a H_∞ -filter. Introducing $\gamma = 1/\lambda$ and

using $R = I$, $D = \mu I$ the Riccati differential equation (6) can be rewritten in the form

$$\dot{P}(t) = A(t)P(t) + P(t)A^T(t) - P(t)C^T(t)C(t)P(t) + \frac{P(t)P(t)}{\gamma^2} + DD^T \quad (34)$$

and the observer gain (7) is given by

$$K(t) = P(t)C^T(t). \quad (35)$$

With (34) and (35) we are able to employ standard results in H_∞ -filtering (cf. [2, Sec. 7.4, p. 301, Th. 7.6] or [6, Sec. 7.2.2, p. 267, Th. 7.2.1]) and conclude that the subsystem inside the dashed box in Fig. 2 is exponentially stable and the operator norm $\|T_{vw}\|_\infty$ is bounded by

$$\|T_{vw}\|_\infty \leq \gamma.$$

Together with (33) and $\gamma = 1/\lambda$ we fulfill the small gain condition (32) and the proof is completed. \square

IV. APPLICATION TO AN EXAMPLE SYSTEM

To examine the practical usefulness of the proposed observer we apply it to a highly nonlinear example system, a symmetrical three-phase induction motor. The quantities to be estimated are the rotor flux and the angular velocity (see also [19] and the references cited therein). We introduce the state differential equations for an induction machine and do not mention too many details about the physical meanings of the considered quantities, which are contained in any standard book about electrical drives (see, e.g., [12, Sec. 10.1] or [15, Sec. 3.3]). The state differential equations of a symmetrical three-phase induction machine read

$$\dot{z}_1(t) = k_1 z_1(t) + x_1(t) z_2(t) + k_2 z_3(t) + x_2(t) \quad (36)$$

$$\dot{z}_2(t) = -x_1(t) z_1(t) + k_1 z_2(t) + k_2 z_4(t) \quad (37)$$

$$\dot{z}_3(t) = k_3 z_1(t) + k_4 z_3(t) + (x_1(t) - z_5(t)) z_4(t) \quad (38)$$

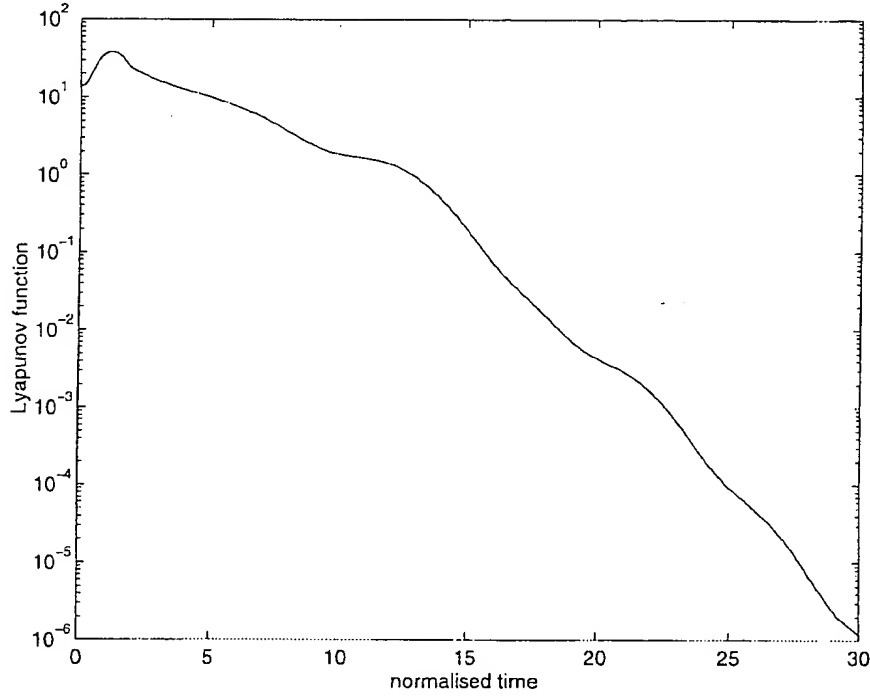


Fig. 4. Lyapunov function $V(\zeta(t), t)$ for the proposed observer.

$$\dot{z}_4(t) = k_3 z_2(t) - (x_1(t) - z_5(t))z_3(t) + k_4 z_4(t) \quad (39)$$

$$\dot{z}_5(t) = k_5(z_1(t)z_4(t) - z_2(t)z_3(t)) + k_6 x_3(t). \quad (40)$$

All state variables are normalized. $z_1(t)$, $z_2(t)$ and $z_3(t)$, $z_4(t)$ are the components of the stator and the rotor flux, respectively, in the plane perpendicular to the rotation axis. $z_5(t)$ is the angular velocity, $x_1(t)$ the frequency, and $x_2(t)$ the amplitude of the stator voltage, respectively, and $x_3(t)$ is the torque of the load. k_1, \dots, k_6 are several parameters depending on the considered drive.

The output are the normalized stator currents $y_1(t)$ and $y_2(t)$, leading to the output equations

$$y_1(t) = k_7 z_1(t) + k_8 z_3(t) \quad (41)$$

$$y_2(t) = k_7 z_2(t) + k_8 z_4(t) \quad (42)$$

with parameters k_7 and k_8 . The matrix $A(t)$ can be calculated from (36)–(40), parameter t omitted

$$A(t) = \begin{bmatrix} k_1 & x_1 & k_2 & 0 & 0 \\ -x_1 & k_1 & 0 & k_2 & 0 \\ k_3 & 0 & k_4 & x_1 - \hat{z}_5 & -\hat{z}_4 \\ 0 & k_3 & \hat{z}_5 - x_1 & k_4 & \hat{z}_3 \\ k_5 \hat{z}_1 & -k_5 \hat{z}_3 & -k_5 \hat{z}_2 & k_5 \hat{z}_1 & 0 \end{bmatrix} \quad (43)$$

and the output matrix is given by (41), (42)

$$C = \begin{bmatrix} k_7 & 0 & k_8 & 0 & 0 \\ 0 & k_7 & 0 & k_8 & 0 \end{bmatrix}. \quad (44)$$

For the simulation we set $k_1 = -0.186$, $k_2 = 0.178$, $k_3 = 0.225$, $k_4 = -0.234$, $k_5 = -0.081$, $k_6 = -0.018$, $k_7 = 4.643$, $k_8 = -4.448$, $x_1(t) = s(t)$, $x_2(t) = s(t)$, and $x_3(t) = 0$, where $s(t)$ is the unit step function. These values are obtained, if in the original unnormalized system certain values for the physical quantities are chosen. To obtain appropriate values for λ and μ we propose a binary search algorithm consisting of the following steps.

1) Choose $\lambda, \mu > 0$ and $R = rI$ with some $r > 0$.

- 2) Decrease r by taking the half-value until the solution $P(\cdot)$ of the Riccati differential equation (6) takes positive definite values. If no positive definite solution can be achieved, proceed with Step 3).
- 3) Decrease μ by taking the half-value and initialize $R = rI$ as in Step 1). Proceed with Step 2). If μ gets smaller than a prescribed bound μ_0 we let the algorithm abort.

For the simulations we insert $A(t)$ and C into the Riccati differential equation (6) and choose $\lambda^2 = 0.7$, $\mu^2 = 0.008$, and $R = 0.06I$ according to the above stated search algorithm. The observer gain is given by (7) and the differential equation for the observer by (3) and (8). We solved the state differential equations numerically by the Runge-Kutta method choosing the initial conditions

$$z(0) = [0.2 \quad -0.6 \quad -0.4 \quad 0.1 \quad 0.3]^T \quad (45)$$

for the system to be observed and

$$\hat{z}(0) = [0.5 \quad 0.1 \quad 0.3 \quad -0.2 \quad 4]^T \quad (46)$$

for the observer. For the sake of comparison we also computed a usual extended Kalman filter [5] with the same weighting matrices $\mu^2 I$, R and the same initial condition $\hat{z}(0)$ by setting $\lambda = 0$, $\mu^2 = 0.008$, and $R = 0.06I$ in (6).

Figs. 3 and 4 show the simulation results. As shown in Fig. 3 the estimation error approaches zero for the proposed observer but not for the extended Kalman filter. Further simulation results have shown that the proposed observer has often a larger domain of attraction than the extended Kalman filter, i.e., the initial estimation error may be larger. The Lyapunov-function (18) is depicted in Fig. 4. Looking at the values for $0 \leq t \leq 1$ we see that the Lyapunov function is increasing. This means that (22) is sufficient but not necessary for the error decay.

V. CONCLUSION

The purpose of this paper is to propose an observer for nonlinear systems. This is done by modifying the well-known extended Kalman filter [5] for an effective treatment of the nonlinearities. We proved that under certain conditions this observer is an exponential observer [8] by choosing an appropriate Lyapunov function. For an alternative approach to the proposed observer we consider the nonlinearities as uncertainties and get a stabilization problem which is typical in robust control theory [9], [20]. This stabilization problem can be solved by H_∞ -filtering techniques [2], [6], [10], [14] leading to the proposed modification of the extended Kalman filter. Using the proposed observer to estimate the rotor flux and the angular velocity of an induction motor we show that it can be easily applied to complex highly nonlinear systems. Numerical simulations show a good performance and an increased domain of convergence in comparison to the extended Kalman filter, i.e., we can tolerate larger initial estimation errors. Similar to H_∞ -filtering problems, we have in general not a bounded positive definite solution of the corresponding Riccati differential equation, which does exist for the extended Kalman filter if the system satisfies certain observability conditions [1]. Therefore, a careful choice of the constants in this Riccati differential equation is of particular interest.

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Fractional-Order Systems and $PI^\lambda D^\mu$ -Controllers

Igor Podlubny

Abstract—Dynamic systems of an arbitrary real order (fractional-order systems) are considered. A concept of a fractional-order $PI^\lambda D^\mu$ -controller, involving fractional-order integrator and fractional-order differentiator, is proposed. The Laplace transform formula for a new function of the Mittag-Leffler-type made it possible to obtain explicit analytical expressions for the unit-step and unit-impulse response of a linear fractional-order system with fractional-order controller both for the open and closed loop. An example demonstrating the use of the obtained formulas and the advantages of the proposed $PI^\lambda D^\mu$ -controllers is given.

Index Terms—Fractional-order controllers, fractional-order systems, fractional differential equations, Laplace transforms, transfer functions.

I. INTRODUCTION

Recently, several authors have considered mechanical systems described by fractional-order state equations [4], [5], [15], which means equations involving so-called fractional derivatives and integrals (for the introduction to this theory see [21]).

There are also several recent applications in electricity.

Le Méhauté and Crepy [14] have proposed a concept of a *fractance*—a new electrical circuit element, which has intermediate properties between resistance and capacitance. Such a device has been experimentally studied, for example, by Nakagawa and Sorimachi [18]. A circuit proposed by Oldham and Zoski [22] provides another example of a fractance.

A new capacitor theory developed by Westerlund [28] is based on the use of fractional derivatives.

New fractional derivative-based models are more adequate than the previously used integer-order models. This has been demonstrated, for instance, by Caputo [7], Nonnenmacher and Glöckle [19], Friedrich

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